MILLIMETER ARRAY
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Imaging of weak sources with compact arrays

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Introduction

Selfcalibration [1] of an array requires that the correlated visibility on most baselines in an array be greater than the noise per antenna in the coherence time [2]. If this is not true then the errors in the final selfcalibrated image will be amplified or, in the worst case, false sources may be manufactured. There appears to be no way to avoid this limit if the phase of an antenna changes completely after one coherence time. However, this is not always so and hence it may be possible to design a selfcalibration algorithms to exploit this fact in the selfcalibration of weaker sources. In this note I will examine the limits on selfcalibration imposed by the physics underlying the calibration problem; I will assume that an algorithm can be designed to approach these limits [see e.g. VLBA memo 25 by Schwab]. The discussion, for reasons which will be apparent, is most appropriate for compact arrays. I will also discuss two alternatives to selfcalibration.

Selfcalibration : some general arguments

Let us consider the most, and least perverse behaviour of the atmosphere above a compact array such as the VLA D-configuration. Worst of all for selfcalibration would be an atmosphere which twinkles, changing the path length above an antenna discontinuously every τ seconds, say. The sequence of path lengths for the array be completely uncorrelated both in space and time. One expects this picture to be appropriate when looking through a phase screen containing many different cells and, indeed, in some senses this decribes the atmosphere above an optical telescope. Selfcalibration fails if the gain error per antenna is still comparable to a radian after correction since coherent integration is not then possible. It is easy to show that for an approximately unresolved source this constrains the flux F such that :

$$F >> \sigma_V/(N-2)^{1/2} \qquad (1)$$

where σ_V is the error in the visibility after τ seconds of integration, and there are N antennae in the array. By comparison, the detection limit for a fully coherent array is :

$$F >> (\sigma_V/N).(\tau/T)^{1/2} \qquad (2)$$

where T is the total integration time.

Best of all would be a frozen atmosphere which merely drifts over the array at some constant velocity. The benefit to selfcalibration depends upon a number of parameters : the characteristic diameter of path length variations in the atmosphere d, the wind speed c, and the configuration of the array relative to the wind, especially the characteristic antenna spacing ℓ. We will describe some of the possible points in this "phase space" in terms of the effective coherence time τ, which we will define as the length of time over which the path lengths can be predicted to better than a radian. To calculate the approximate flux limit after selfcalibration one simply inserts the relevant σ_V for that coherence time into equation (1). For a well filled, two dimensional array of total diameter L :

$$d \ll \ell : \tau \simeq d/c$$

$$d \simeq \ell : \tau \simeq L/c$$

$$d \gg \ell : \tau \simeq d/c$$

In the case of a one dimensional array the wind direction is important.

$$d \ll \ell : \tau \simeq d/c$$

$$d \simeq \ell, \text{ wind } \parallel \text{ array} : \tau \simeq L/c$$

$$d \simeq \ell, \text{ wind } \perp \text{ array} : \tau \simeq d/c$$

$$d \gg \ell : \tau \simeq d/c$$

Thus, in the most favourable circumstances the effective coherence time is increased by a factor L/d and the flux limit (viz. equation (1)) is decreased by a factor $(d/L)^{1/2}$. Since the VLA configuration falls somewhere between these two extremes, one would expect that at least one arm of the Y would make such a large angle to the wind that selfcalibration would be difficult. The problem of whether this simple frozen screen picture is appropriate for the millimetre array can only be settled by experiment.

We do have some information about the atmosphere. Sramek (1983) has made λ1.3-cm D-conf. observations of calibrator sources. The main finding is that the structure function of the phase errors is, on average :

$$D(r) = 14.r^{0.36} \text{ degrees at 22 GHz.} \qquad (3)$$

where r is the baseline length in kilometers. To orient ourselves note that this corresponds to a phase error of one radian on a one kilometer baseline at 90 GHz. This can be translated into the temporal domain if we assume that the frozen screen model applies (Sramek shows some evidence that this may be the case : the r.m.s. phase error on a baseline of 1Km seems to increasely approximately linearly with wind velocity). If v is the wind velocity :

$$D(t) = 14.(v.t.\sin(\theta))^{0.36} \text{ degrees at 22 GHz. (4)}$$

To illustrate the relative importance of receiver phase noise and atmospheric phase noise I have plotted these quantities in Figure 1 as functions of wind integration time, wind velocity and source flux. The approximate sensitivity of the mm. array will be about 0.3 radians of phase error per antenna for a one Jansky source in one second. The optimum integration time is such that the phase errors are equal. It is clear that long baseline observations at short wavelengths will only be possible on very good days e.g. 1Km phase at 22Ghz < 5-10 degrees. One simple and particularly illuminating observation concerns the correlation of the ANTSOL solutions for antenna pairs having the same separation but lying on different arms of the Y; we hope that the peak correlation would be greatest for the arm pointing closest to the wind direction. Note also that the examination of the lags at which the correlation functions peak on each arm should triangulate the wind vector reasonably well.

It should be emphasized that these speculations delineate the range of possibilities for selfcalibration of the millimeter array. Questions of algorithms and practicability are very dependent upon what we learn about the atmosphere.

Without selfcalibration ?

Even if the effective coherence time is still too short to allow a particular source to be selfcalibrated it will still be possible to integrate only the amplitude information. Fourier transforming the square of the amplitude will yield the autocorrelation of the true image, convolved with the dirty beam (in that order). We could then use Fienup's algorithm [3] to restore the phase and to deconvolve. For simple sources there should be enough constraints to allow a reasonably accurate reconstruction. Of course, the noise only decreases with the fourth root of the number of independent visibility samples (i.e. after as much coherent integration as possible) but, in principle, imaging of arbitrarily weak sources is possible. The flux limit for this type of processing is :

$$F >> (\sigma_V/N^{1/2}).(\tau/T)^{1/4} \qquad (4)$$

Constraints upon T come mainly from the telescope scheduling committee. This approach is closely related to speckle interferometry but we have the luxury of direct access to the visibility, rather than the Fourier transform in which redundant spacings are intermingled. Consequently averaging over many speckles is not necessary except to beat down the noise.

Imaging from the intensity only is probably not of much interest since the signal-to-noise only increases with the fourth root of the number of coherent integrations. Thus to improve the signal to noise by a factor 10 we have to observe for 10,000 coherence times as opposed to 100 coherence times if phase information can somehow be used. It seems therefore that improving selfcal by a relatively small factor will produce a better payoff.

Seeing disks

Imaging of arbitrarily weak sources is, of course, possible with optical telescopes much larger than the atmospheric coherence patch. Each coherence patch acts as an independent telescope with its own resolution and collecting area, and the images formed by these telescopes are added to produce the overall image. Thus both resolution and sensitivity are degraded below that obtained in the absence of phase errors.

F. Owen has suggested that the seeing disk concept may be applicable to compact radio arrays. We will now examine this point. It is easily shown that the seeing disk is the Fourier transform of the exponential of (minus one halfth) the structure function. For an unfilled aperture the seeing disk is modified by convolution with the point spread function of the aperture. Hence, roughly speaking, only antennae in the same coherence patch can be used together coherently. Correlations with antennae outside the coherence patch merely add in noise and thus should be discarded. It should then be clear that, given a fixed number of antennae, the optimum configuration has all antennae in the same coherence patch. The resolution is then the same but the sensitivity is greater since all antennae add coherently.

Summary

The most basic question concerning the selfcalibration of weak sources concerns the predictability of the phase variations over the array. The effective coherence time is that interval for which the phase variations can be reliably predicted; this, in turn, constrains the weakest source that can be selfcalibrated. Filling in these numbers requires some more experimental work. For sources too weak for selfcalibration reconstruction from the amplitudes alone is feasible, if time consuming. Finally, it should be noted that the possibility of low signal to noise selfcalibration must affect the design of compact arrays.

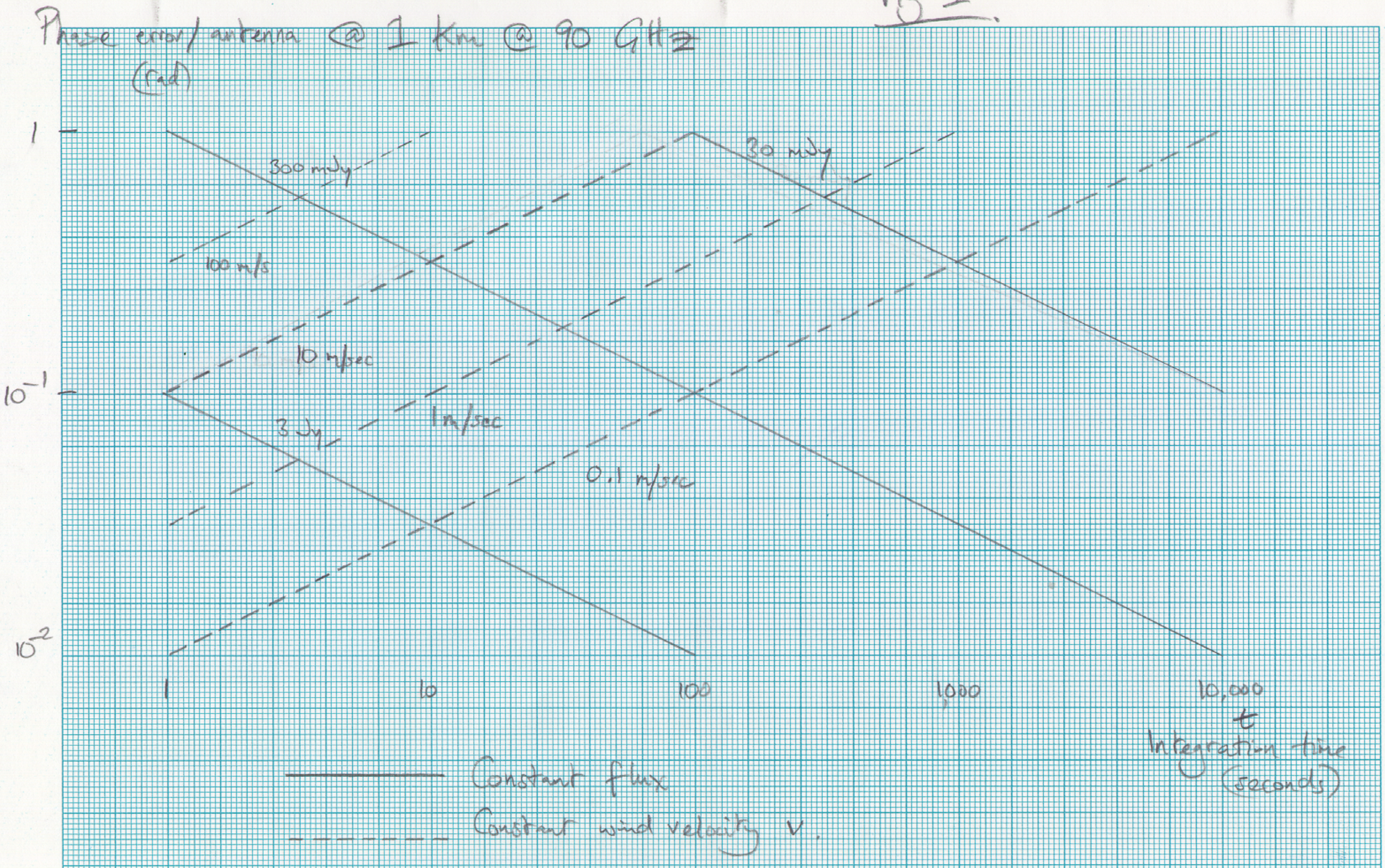
References

[1] Schwab,F.R., "Adaptive calibration of radio interferometer data", SPIE, 231,18-24, 1980.

[2] Cornwell,T.J., "An error analysis of calibration", VLA scientific memorandum 135, 1981.

[3] Fienup,J.R., Opt. Lett., 3,27-29, 1978.

Fig 1



$$\sigma_{noise} \sim \frac{0.3}{F\sqrt{t}} \qquad \sigma_{atmos} \sim \sqrt{\frac{vt}{1000}}$$