MILLIMETER ARRAY
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# Brightness temperature limits for filled and unfilled apertures

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## Introduction :

In MM array memo 18 I suggested a method of quantifying the surface brightness sensitivity of a possible configuration. As a result of subsequent discussions I now think that some elaboration and simplification of the concepts alluded to in that memo would be worthwhile. In particular, the connection between the surface brightness sensitivity of unfilled and filled apertures should be elucidated.

Before launching into some maths we should establish what we expect to learn. The surface brightness sensitivity of a telescope can best be described by the minimum brightness temperature which it can detect. For a filled aperture this is :

$$T_{B,filled} = T_{sys}/(\eta.(B.\tau)^{1/2})$$

where $T_{sys}$ is the system temperature, $\eta$ is the effective efficiency of the telescope, B is the bandwidth and $\tau$ is the observing time. If the source of interest occupies only a fraction of the beamwidth then beam dilution occurs, and so $T_{B,filled}$ rises to :

$$T_{B,filled} = (T_{sys}/(\eta.(B.\tau)^{1/2})).(\Omega_p/\Omega)$$

where $\Omega_p$ and $\Omega$ are the solid angles of the primary beam and source.

We wish to understand how the minimum surface brightness for an array of N elements is related to the limit for an individual element. An unfilled aperture has some advantage over the filled aperture since we can effectively change the illumination to suit the object being studied. For example, spacings sensitive to scale sizes much smaller than the source only contribute noise and thus should be neglected. Such adaptive weighting is practicable in the unfilled aperture in which individual spacings are measured. However, even for an optimally weighted set of visibility samples we expect that the effective system temperature should be increased from the value $T_{sys}/N$ by a factor representing the fraction of spacings which must be weighted down. It is this effect that we wish to estimate.

Array efficiency :

Brightness temperature is defined by the equation :

$$T_b = \lambda^2.S/(2.k.\Omega)$$

where $\lambda$ is the wavelength of observation and S is the flux density coming from a solid angle $\Omega$. Hence, to get the limit in $T_b$, we need to calculate the r.m.s. error in the determination of the flux of a source of known solid angle. In MM memo 18 I showed that the r.m.s. error in the estimation of the flux density of a circular Gaussian is :

$$\sigma_{map}(\theta) = \sigma_v/(\Sigma_i \exp(-2.\alpha.(r_i.\theta/\lambda)^2))^{1/2} \quad \text{Jy.}$$

where $r_i$ is the radius of the $i^{th}$ sample in meters, $\theta$ is the HPBW width of the Gaussian in radians, $\lambda$ is the observing wavelength in meters, and $\alpha$ is a constant, 4.ln(2). The units of this were mistakenly written as Jy/beam rather than Jy. This formula can be derived from considering a least squares fit of a circular Gaussian to the visibility data. Simple error propagation yields this equation if one assumes that the Gaussian is at a known position. The factor of two in the exponent is present because an optimum taper, matching the source size, is appropriate.

Combining all these equations appropriately, and evading factors of order unity arising from definitions of scale size, one can show that the brightness limit for an unfilled aperture formed from N antennae is :

$$T_{B,unfilled} = (T_{B,filled}/N).(N^2/(N.(N-1)))^{1/2}.\Gamma^{-1}$$

Each of the three terms of the right hand side of this equation can be ascribed to an intuitively obvious effect :

(1) The first term represents the improved sensitivity available from N antennae.

(2) The second term involving the number of antennae is needed since interferometer arrays ignore the autocorrelation terms.

(3) The third term, involving $\Gamma$, results from diminishing signal to noise on longer spacings for a resolved source. $\Gamma$ can be regarded as the efficiency of the array when observing an extended source.

$$\Gamma = ((\Sigma_i \exp(-2.\alpha.(r_i.\theta/\lambda)^2))/N_{vis})^{1/2}$$

where there are $N_{vis}$ visibility samples.

For a continuous distribution of samples the summation should be replaced by an integral. Together with the factor $\Omega_P/\Omega$ in $T_{filled}$, $\Gamma$ contains all the array dependent scaling information about the behaviour of surface brightness sensitivity.

<u>Some Examples :</u>

As an example of the use of this equation for the brightness temperature limit, we consider the surface brightness sensitivity of a simple array in which the number of u,v samples is constant out to a cutoff radius $r_{max}$. It is convenient to introduce characteristic scale sizes in angle such that $r_{max}.\theta_{min} = \lambda$ and $r_{min}.\theta_{max} = \lambda$. Then for small

$\theta/\theta_{min}$, the array efficiency $\Gamma$ is approximately unity, and $T_{unfilled}$ varies simply as the beam dilution of the source in the primary beams of the individual antennae. For intermediate $\theta/\theta_{min}$, the array efficiency starts to drop as $(\theta/\theta_{min})^{-1}$ since not all baselines have high SNR. Therefore, $T_{unfilled}$ goes as $(\theta/\theta_{min})^{-1}$. For large $\theta/\theta_{min}$, the efficiency is very poor, and is dominated by the SNR on the shortest spacing. Then, $T_{unfilled}$ goes as $(\theta/\theta_{min})^{-1}.\exp(\alpha.(\theta/\theta_{max})^2)$.

As another example, consider an array with a Gaussian distribution of u,v points, optimum for a source of HPBW $\theta_A$ :

$$n(r) = A.\exp(-\alpha.(\theta_A.r/\lambda)^2)$$

for $r_{min} < r < r_{max}$. Then we have that the array efficiency is :

$$\Gamma = (1+(\theta/\theta_A)^2)^{-1/2}.(((1-\exp(-2.\alpha.(\theta^2+\theta_A^2).(\theta_{min}^{-2} - \theta_{max}^{-2})))$$
$$/(1-\exp(-2.\alpha.\theta_A^2.(\theta_{min}^{-2} - \theta_{max}^{-2})))^{1/2}$$

Thus the efficiency drops off rather less rapidly with increasing source size than that for a "flat" array. Consequently, the brightness temperature limit is smaller for intermediate source size.

Summary :

The minimum brightness temperature detectable with an array of antennae varies with the source size in two simple ways : first, as the inverse of the source solid angle because of standard beam dilution, and second, as the inverse of an efficiency, which drops off from unity as the source size increases and fewer baselines have good SNR.

Finally, a complete form for the brightness temperature limit is :

$$T_{B,unfilled} = (T_{sys}/(N.\Gamma.\eta.(B.\tau)^{1/2})).(\Omega_p/\Omega).(N^2/(N.(N-1)))^{1/2}$$

where the efficiency $\Gamma$ is :

$$\Gamma = ((\Sigma_i \exp(-2.\alpha.(r_i.\theta/\lambda)^2))/N_{vis})^{1/2}$$