MILLIMETER ARRAY
MEMO NO. 44

# The size of the central element: pointing considerations

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## INTRODUCTION

The question of the optimum size of the central element for the millimeter array is quite complex, and depends upon many different factors such as pointing stability, degree of knowledge of the primary beams of the antennas, cross-talk between the antennas, and so on. In this note, I will attempt to analyze the relevance of the first factor, pointing. This will be based upon my recent memo on the Ekers-Rots scheme of short-spacing estimation (Ekers and Rots, 1979), which is closely related to the imaging scheme which the array will use, namely mosaicing (Cornwell, 1984). The attraction of the Ekers-Rots scheme is that it is more easily investigated than the mosaicing method; a simple one-dimensional analysis is presented in millimeter memo number 42. In this memo I will apply that analysis to single-antenna observations.

## ANTENNA SIZE

Let us suppose that a measurement of a given spacing, $\xi$ meters, is required. We can use a single antenna of diameter $d > \xi$ to find this spacing: we point the antenna at a number of different locations and transform with respect to the pointing offset (see memo 42 for more details). The variance in the derived visibility induced by pointing errors is:

$$\sigma_i^2(u,\xi) \sim \frac{\pi^2}{N_p}\left|\frac{a(0)}{a(\xi)}\right|^2 \left(\frac{\sigma_{x_p}}{\Delta x_{p_{crit}}}\right)^2 S^2 \left(\frac{\lambda\xi}{d}\right)^2 \qquad (1)$$

where $\sigma_{x_p}$ is the r.m.s. pointing error, $a(\xi)$ is the transform of the primary beam, $S$ is a measure of the source flux, $\lambda$ is the observing wavelength, $d$ is the antenna diameter, $\Delta x_{p_{crit}}$ is the critical sampling rate in the image plane, and $N_p$ is the number of pointings required to span the object.

This is a simple approximation to a much more complicated expression but it is adequate for our purposes. It assumes that useful information on a point source is obtained even when it is sitting the far sidelobes of the primary beam. Therefore, this equation is only relevant for fairly small numbers of pointings.

The most important features of this equation are:

1. The error declines as the inverse square root of the number of pointings. This arises because each pointing error is assumed to be independent. Correlated errors will reduce the effective value of $N_p$, but will be more easily estimated via self-calibration techniques.

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2. The error increases drastically since $\xi \rightarrow d$ as the illumination drops off and the leverage of any pointing error increases.

We are interested in the variation of this error as the ratio $\frac{\xi}{d}$ changes. The question is: for a given $\xi$ at which we wish to find the visibility, how big must we make $d$? If we make $d$ too big, then pointing errors will grow, whereas if we make $d$ too small then there will be few measurements at spacing $\xi$ i.e. $a(\xi) \ll a(0)$. To simplify further, assume uniform illumination:

$$a(\xi) = a(0) \left(1 - \frac{|\xi|}{d}\right) \tag{2}$$

and that the pointing errors scale as the $\alpha$ power of the antenna diameter:

$$\sigma_{x_p} \sim d^{\alpha} \tag{3}$$

Furthermore, we have that:

$$\Delta x_{p_{crit}} = \frac{\lambda}{2d} \tag{4}$$

and:

$$N_p = \frac{\Theta}{\Delta x_{p_{crit}}} \tag{5}$$

where $\Theta$ is the angular extent of the object.

If we measure $d$ in units of $|\xi|$, $D = \frac{d}{|\xi|}$, then the error induced by pointing errors is proportional to a function $f(D)$:

$$f(D) = \frac{D^{\alpha+\frac{1}{2}}}{D-1} \tag{6}$$

which has a minimum at:

$$D_{opt} = \frac{\alpha + \frac{1}{2}}{\alpha - \frac{1}{2}} \tag{7}$$

This function is plotted in figure 1 for various values of $\alpha$.

Some interesting conclusions are immediately apparent:

- If the pointing errors are independent of antenna diameter, i.e. $\alpha = 0$, then $D_{opt}$ is infinite. Since there is no penalty in increased pointing errors, one can increase the antenna diameter without limit. This is also the case for any $\alpha \leq \frac{1}{2}$.
- For the case of pointing errors proportional to antenna diameter, $\alpha = 1$, the optimum antenna diameter is 3 times the desired spacing.
- For sharper dependencies of the pointing errors on antenna diameter, i.e. $\alpha \gg 0$, the optimum antenna diameter moves closer to the desired spacing. For example, for $\alpha = 2$, $D_{opt} = \frac{5}{3}$, and for $\alpha = 3$, $D_{opt} = \frac{7}{5}$.
- For weak dependencies upon the antenna diameter, the curve is quite flat in the neighbourhood of the minimum. For example, for $\alpha = 1$, $f(D)$ is no more than 40% higher than the optimum value for any diameter in the range $1.5 < D < 11$.

## Discussion

We can use these results to investigate the problem of filling in the shortest spacings. If we were to use a large single-dish to fill in the gaps between interferometer elements, then we would want to get good measurements at $\xi$ equal to the diameter of the interferometer elements. We can therefore interpret $D_{opt}$ (equation 7) as the optimum value of the ratio of Central Element diameter to interferometer element diameter. For example, if $\alpha = \frac{3}{2}$ then the Central Element should be a 15 meter diameter dish (assuming that the interferometer elements have the canonical diameter of 7.5 meter).

In the case of a homogeneous array (i.e. all array elements having the same size), we need to mosaic to get spacings up to $\frac{d}{2}$ from the total power measurements, and down to $d - \frac{d}{2}$ from the interferometer measurements. Therefore $D = 2$, which is optimum for $\alpha = \frac{3}{2}$, and only 8% worse than optimum for $\alpha = 1$. This is further illustrated in figure 2, where I show the ratio of $f(D_{opt})$ to $f(2)$ as a function of $\alpha$. This ratio can be interpreted as the SNR loss of the homogeneous array relative to the use of an optimum special-purpose central element. For $\alpha$ in the range $0.5 \rightarrow 4$ this loss is no more than a factor of two, and therefore, as far as pointing errors are concerned, the homogeneous array is acceptable for any case where $\alpha < 4$.

We should remember that this analysis is one-dimensional, and is approximate. Fuller answers must come from simulation.

## References


Cornwell, T.J. (1984), *NRAO mm array memo 24.*

Cornwell, T.J. (1987), *NRAO mm array memo 42.*

Ekers, R.D. and Rots,A.H. (1979), *Short Spacing Synthesis from a Primary Beam Scanned Interferometer*, Proc. IAU Col. 49, Image Formation from Coherence Functions in Astronomy, Ed. C. van Schooneveld, D. Reidel, 61-66.

f(D) for various values of α

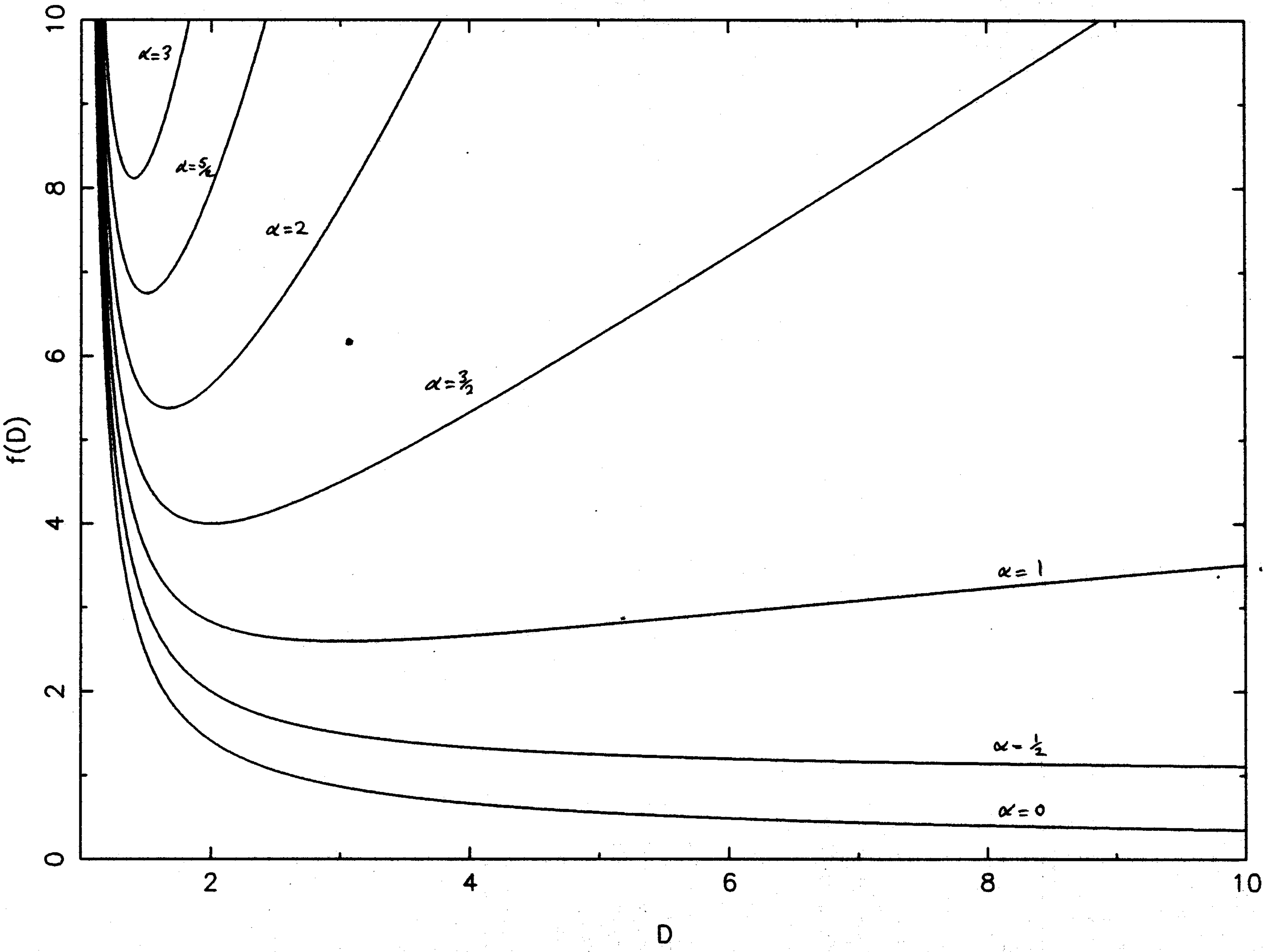
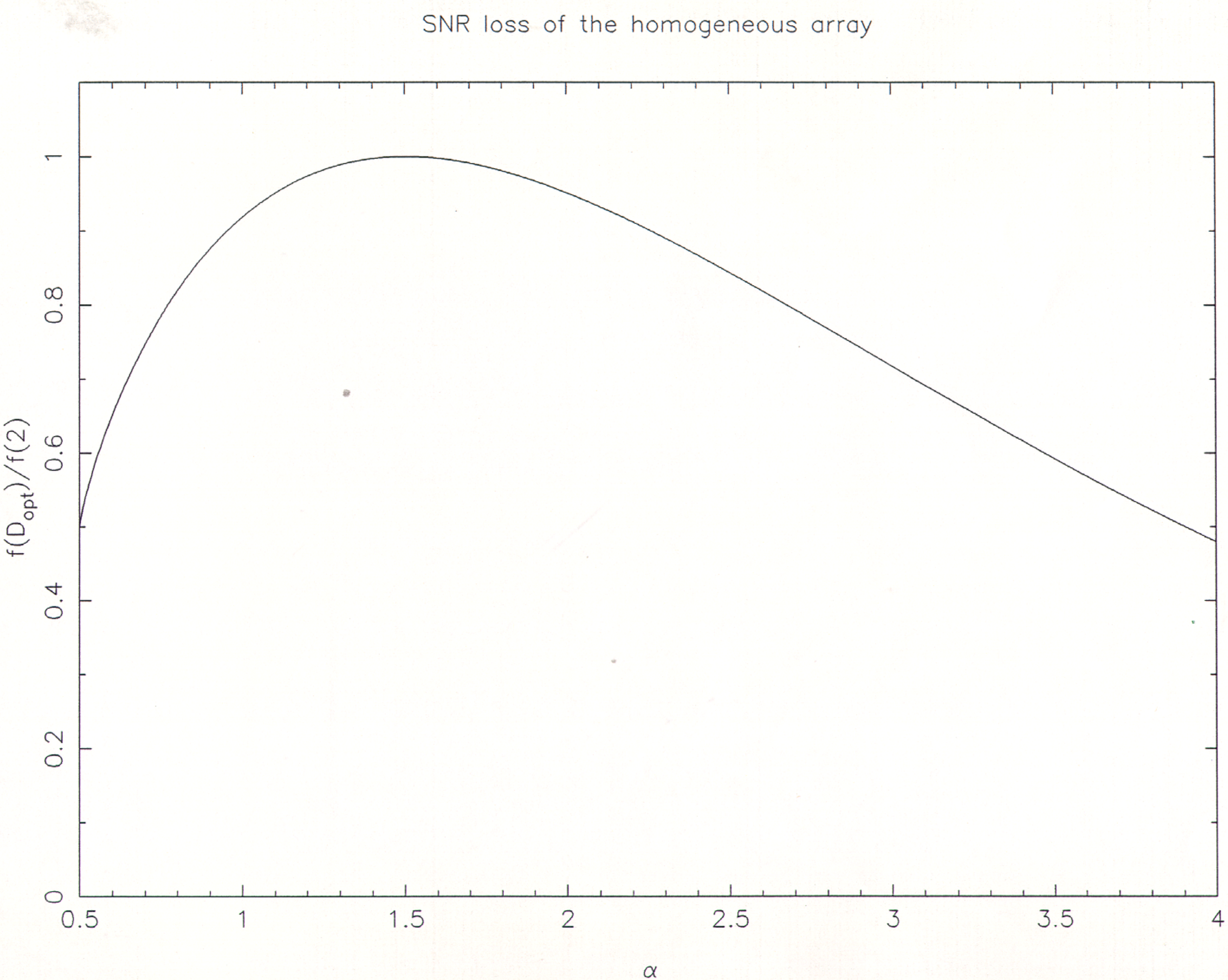



Figure 1



SNR loss of the homogeneous array
f(D_opt)/f(2)
0
0.2
0.4
0.6
0.8
1
0.5
1
1.5
2
2.5
3
3.5
4
α


Figure 2