MILLIMETER ARRAY
MEMO NO. 55

Millimeter Array Correlator Cost Equation

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In reviewing the Correlator section of the design study (*Millimeter Array Design Concept*, section VI.4, NRAO, Jan. 1988), I noticed several minor errors in the cost equation, as follows:

1. The output of the analog filters is a real signal, and sampling at the rate $2B/J$ will produce enough data for only B/JK length-K *complex* FFTs per unit time, not $2B/JK$. Assuming that the factor C_T is the cost of a complex butterfly per unit time, the second term in the cost equation is too high by a factor of 2.

2. The above error carries over to the third term, in that there are only half as many products required per unit time. But these are now full complex products, requiring 4 real multiplies and 4 real adds each. The factor N^2 for the number of *real* correlators per lag (assuming that C_X refers to the cost of a real correlation per unit time) follows from assuming two real correlators per product (sine and cosine or positive and negative lags). Thus, the two errors cancel and the third term is correct.

3. The third term does not account for the additional cross correlations needed if polarization is to be measured. This can be handled by assuming, in accord with our usual practice, that the spectral resolution or bandwidth will be degraded by a factor of 2 when full polarization is required, so that the rate of cross correlation remains the same. Notice that polarization measurement constrains the number of filters J to be even.

It is also useful to insert a filter shape factor β as was done by Weinreb in EDIR #248. This is the factor (ratio of non-overlapped bandwidth to Nyquist sampled bandwidth) by which the analog filter bandwidths will overlap in order to construct a smooth spectrum. Letting the final spectral resolution be b (to the $2/\pi$ points), the cost equation becomes

$$C_{tot} = C_F\,N\,J + C_T\,N\,\beta B\,\log_2(\beta B/bJL) + C_X\,2N^2L\beta B,$$

where $\beta B/bJL = K$ is the FFT length. Correction 1 above also requires that the optimum values of L and J be reduced by factors of 2:

$$L_{opt} = 0.721\frac{C_T}{NC_X}$$

$$J_{opt} = 1.442\frac{C_T\beta B}{C_F}.$$

There are a few problems with the linear optimization that yields the latter two equations. First, the parameters L, J, and K are constrained to be positive integers. If we find $L_{opt} \ll 1$ formally, as happens in practical cases, then rounding up to $L = 1$ will cause C_{tot} greatly to exceed its formal minimum. Next, for convenient FFT implementation, K should be highly composite, preferably a power of 2, and must not be too small; $K \geq 32$ is a reasonable constraint. Finally, the bandwidth may be filtered into several channels for reasons independent of this optimization (e.g., dual band operation, bandwidth synthesis, polarization), forcing additional constraints on J.

The cross correlation cost rate C_X deserves special attention, in that it depends strongly on the dynamic range and organization of the numbers to be correlated. In the case $F = 1$

(no FFTs, an "XF" correlator), the data is coarsely quantized (1 or 2 bits) and each cross multiplier can always be associated with the same integration register; furthermore, the data is real, simplifying the multiplier organization (although the real correlation rate per frequency channel is the same as for a complex correlator). Otherwise (for an "FX" correlator), the potential dynamic range increases in proportion to $\log K$; successive products must be accumulated in separate registers (because they represent different frequencies or baselines); and the numbers are complex. The net result is that C_X is much larger in the latter case. For example, I estimate that an XF correlator using the NFRA chip (2-bit real signals) would yield $C_X = C_{X1} = 110\,\$/\mathrm{GHz}$, whereas an FX correlator using the VLBA chip (5,5,4 complex floating point signals) would yield $C_X = C_{X2} = 2160\,\$/\mathrm{GHz}$. The VLBA chip is designed primarily to compute FFT butterflies, so the same technology might yield a lower cost factor if tailored to the cross-correlation task.

The value of C_X used in the design study was 500 \$/GHz. This is based on the VLA correlator, which is an XF type with 3-level signals. The use of more modern technology would reduce this (e.g., to the estimated 110 \$/GHz mentioned above), and it was thought that this would compensate for the extra complexity of the cross correlations required with the FX architecture. It now appears that the cost of the extra complexity is much larger than this. Using the values of C_{X1} and C_{X2} given above (details of their derivation to be given in a later memo), the costs in Table 1 of the design study (page 64) become:

J	K	L	C_{tot}	
14	72	1	\$16,400k	Optimum FX, C_X=2160 \$/GHz
128	1	8	10,240k	Optimum XF, C_X=110 \$/GHz
8	128	1	16,460k	FX with minimum practical J
8	1	128	90,400k	XF with minimum practical J

The last case has been added for comparison. In all these cases, $N = 40$, $B = 2\,\mathrm{GHz}$, $b = 2\,\mathrm{MHz}$, $C_F = 900\,\$$, and $C_T = 4200\,\$/\mathrm{GHz}$. I took $\beta = 1$ for simplicity. To find the optimum XF correlator, C_{tot} is minimized under the contraint $K = 1$; in that case,

$$J_{opt}(XF) = \beta B \sqrt{2NC_X/bC_F}.$$

The FX costs in the table are dominated by the cross-multiplication term (85%); formally, $L_{opt} = .035$, so rounding forces this term to be 28.5 times the formal minimum.

It might seem surprising that the XF correlator appears cheaper for the MMA. For the VLBA case ($B = 128\,\mathrm{MHz}$, $B/b = 8192$, $N = 20$), using the same cost factors, one finds that FX wins by a wide margin. To gain some insight into the dependencies, consider the formal optimum cost obtained by inserting $J_{opt}, K_{opt}, L_{opt}$ into the cost equation:

$$C_{opt}(FX) = N\beta BC_T \left[2.884 + \log_2\left(\frac{NC_FC_{X2}}{1.040{C_T}^2 b}\right)\right],$$

$$C_{opt}(XF) = 2N\beta B\sqrt{2C_{X1}C_FN/b}.$$

Notice that the optimum cost is just proportional to the total bandwidth in both cases. The antenna number and frequency resolution dependencies are weaker for the FX ($\propto N \log N/b$) than for the XF ($\propto N\sqrt{N/b}$), so the FX will win for sufficiently many antennas or small resolution, other things being equal. I do not see a simple way to describe the cost factor dependencies, so each case must be evaluated separately. Remember also that these formal optima may not be achieved when rounding and other constraints are considered.