MILLIMETER ARRAY
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Geometry Of The Slant-Axis Antenna

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Abstract

The geometry of a slant-axis antenna is described, and pointing equations are given which relate the rotations at the bearings of the antenna to the hour angle and declination of a source. A comparison is made with a conventional altazimuth mount. The requirements for fast switching through small angles on the sky are also discussed. It is found that the main difference is at low elevation angles where the slant-axis antenna needs to rotate much further for a given angle on the sky.

1 Introduction

The slant-axis antenna (Fig. 1) has been suggested as a possible design for the mmA antennas independently to James Lamb by Dietmar Plathner (IRAM), and to John Payne by N. L. Hanson (Daytron Systems) because of its greater stiffness than the conventional altazimuth antenna. It has a conventional azimuth bearing, but the elevation motion is produced by rotating about an axis which is not horizontal. There are several areas where the slant-axis antenna may be superior to the altazimuth antenna:

- Greater stiffness, leading to better servo performance
- Better resistance to wind turning moments
- Larger receiver and optics accommodation
- Smoother outline giving lower wind resistance and better weather protection
- No need for counterweight

These advantages need to be definitively demonstrated and a detailed comparison of a slant-axis design with a more conventional altazimuth configuration is being undertaken—this note concentrates on the geometrical effects of the non-orthogonal axes.

2 Geometry

The slant-axis design is a generalization of the standard altitude over azimuth (altazimuth) configuration. It is shown in Fig. 1*a* with two defining parameters, the bearing tilt, α and the optical-axis offset angle, β. To be able to point to the zenith, these two angles must be the same, $\alpha = \beta$. Since this is a requirement for the mmA this will be assumed in the rest of this note and α will be used to denote both angles. In the standard altazimuth mount both angles are 90°. If $\alpha = 45°$ the antenna may point to any elevation between the horizon and the zenith. However, the change in angle of the upper bearing for a given elevation change, $d\theta'/d\theta$, tends to infinity at the horizon, though this can be avoided by increasing α Most astronomical observations will be made at elevations at least 5° above the horizon where the problem is not very severe. Ground based holography may require lower elevations, but the transmitter should be located above zero elevation anyway to minimize ground reflections.

A further generalization can be introduced by tilting the azimuth bearing also. This avoids the gimbal lock at the zenith but moves it to some other point (along the axis of the azimuth bearing). A particular case of this is a polar mounted telescope which avoids the gimbal lock altogether. For the present it will be assumed that the azimuth bearing axis is vertical.

The non-orthogonality of the axes leads to some complications in the drive system, but it is not anticipated that this will be a major problem. It is not strictly possible to determine the torque and power requirements simply by looking only at the initial and final positions (when the antenna is being switched from one position to another, for example), but it may be seen that when the elevation of the slant-axis antenna is changed the surface of the primary is rotated about its axis and the structure between the two bearings is also rotated, in contrast to the conventional altazimuth design. The moments of inertia about these axes are probably relatively small and the increased torque requirements will not be excessive.

3 Pointing

The pointing vector of the antenna may be understood with by referring to Fig. 1*b*. The rotations about the azimuth and slanted bearings are ψ' and θ', respectively. $\boldsymbol{p}$ is the pointing vector of unit length that can also be described by a conventional azimuth angle, ψ, and elevation angle, θ. As the upper bearing is rotated through 360° vector $\boldsymbol{p}$ sweeps out a cone of half angle β, resulting in a change of both azimuth and elevation. To move the antenna in elevation only, both bearings need to be rotated, while only the lower bearing needs to be turned for an azimuth change. Elevation and azimuth angles are given (for $\alpha = \beta$) by

$$\sin(\theta) = \cos(\alpha)^2 - \sin(\alpha)^2 \cos(\theta') \quad (1)$$

and

$$\tan\psi = \frac{\cos\psi'\sin\theta' - (\cos\theta' + 1)\cos\alpha\sin\psi'}{\sin\psi'\sin\theta' + (\cos\theta' + 1)\cos\alpha\cos\psi'} \quad (2)$$

A source can be tracked by varying θ' and ψ' appropriately. Fig. 2*a* and Fig. 2*b* respectively show how these two angles vary while tracking sources at different declination angles over a range of hour angles (from rising to setting). It is assumed that the site latitude is 34.1°. Fig. 3*a* and Fig. 3*b* show the same quantities for an altazimuth mount. As expected the variation in θ' is symmetric in hour angle in both cases. ψ' has odd symmetry for the conventional mount, but no symmetry for the slant-axis mount. In the latter case, the trajectory is made up of a part with odd symmetry from the change in azimuth of the source, and a part with even symmetry due to the compensation of the azimuth changes induced by the rotation of the slant-axis. All the curves assume that $0° \leq \theta' \leq 180°$ but a corresponding set could be obtained with $180° \leq \theta' \leq 360°$, in which case the azimuth curves would be mirror images.

4 Small-Angle Switching Properties

One of the proposed observing modes for the array is to observe an astronomical source and to switch to a nearby calibration source at short time intervals to calibrate the atmospheric phase. We do not anticipate that the drive will be a problem for normal tracking, but this rapid switching between nearby sources on the sky will place a heavy demand on the servo system. Switching in azimuth will be similar for the conventional and slant-axis geometries, but movements with an elevation component will be different since both bearings of the slant-axis antenna need to turn. Consider a requirement for switching through some small angle δ on the sky in a direction at an angle γ to the horizontal (Fig. 4). In terms of the change in elevation, and azimuth

$$\delta^2 = \Delta\theta^2 + \Delta\psi^2 \sec^2\theta \tag{3}$$

and

$$\tan\gamma = \frac{\Delta\theta}{\Delta\psi\sec\theta} \tag{4}$$

The rotations required of the two bearings are related to the azimuth and elevation changes by

$$\Delta\theta' = \frac{\partial\theta'}{\partial\theta}\Delta\theta + \frac{\partial\theta'}{\partial\psi}\Delta\psi \tag{5}$$

and

$$\Delta\psi' = \frac{\partial\psi'}{\partial\psi}\Delta\psi + \frac{\partial\psi'}{\partial\theta}\Delta\theta \tag{6}$$

The partial derivatives may be found analytically and the following expressions give the bearing rotations in terms of δ and γ.

$$\frac{\Delta\theta'}{\delta} = \frac{-\cos\theta}{\sqrt{\cos^2\theta + 2(\sin\theta - 1)\cos^2\alpha}}\sin\gamma \tag{7}$$

and

$$\frac{\Delta\psi'}{\delta} = \frac{\cos\alpha\,(\sin\theta - 1)}{\cos\theta\sqrt{\cos^2\theta + 2(1 - \sin\theta)\cos^2\alpha}}\sin\gamma - \frac{\cos\gamma}{\cos\theta} \tag{8}$$

Fig. *5a* and Fig. *5b* show how the required bearing rotations depend on the elevation of the source. For the conventional antenna (Fig. *5b*) the maximum angle that the elevation bearing turns is the same as the angle between the source and the calibrator and occurs, of course, when $\gamma = 90°$. The azimuth bearing rotation depends on the elevation of the source and large angles are needed when $\gamma = 0°$ at high elevations. The slant-axis antenna (Fig. *5a*) behaves has a very similar behavior for its azimuth axis (in fact, it is slightly worse at low elevations where the maximum rotations occur for $\gamma = 26.5°$). At low elevations the 'elevation' axis needs to move further than δ, and the increase in angle is similar to that required for the azimuth axis at high elevations.

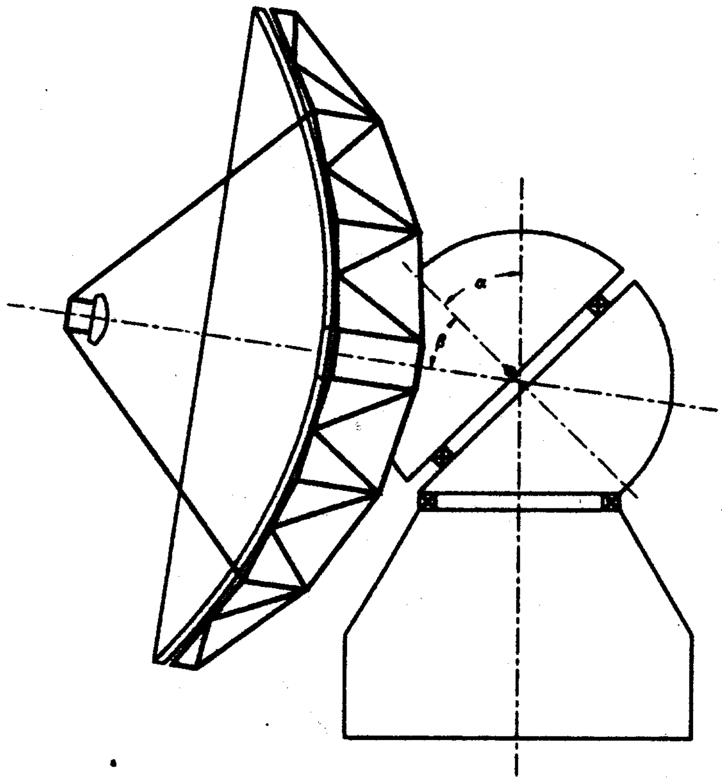
These results show that both axes in the slant-axis antenna need to be driven further than the angle between the source and the calibrator, but the worst cases occur at the opposite elevation extremes. Furthermore, the magnitude of the effect is no worse than for the conventional antenna, although that is affected in only one axis and at one elevation extreme.

5 Discussion

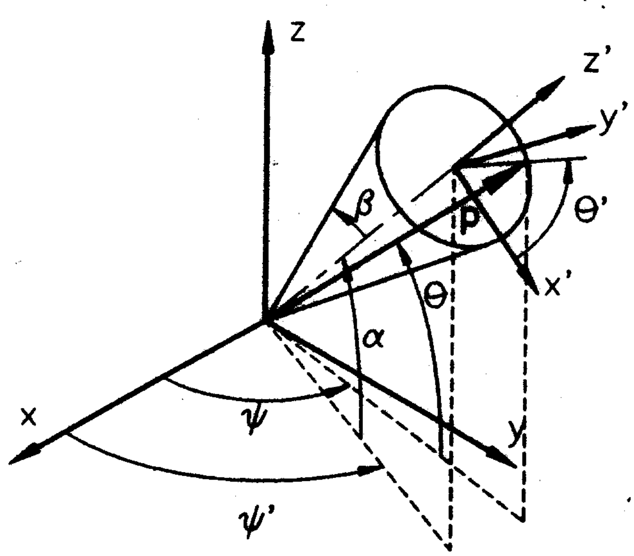
The slant-axis antenna is more complicated in geometry than the conventional altazimuth one. Generally the axes of the slant-axis antenna need to be rotated further than those of the conventional design (though the moments of inertia may be smaller round those axes). Complete sky coverage is obtained if the slant bearing rotates between 0° and 180°, but if it is also permitted to rotate between 180° and 360° lower rotation rates for the azimuth bearing could be achieved for some parts of the sky. If the receivers are located below the slant axis there will be only a small number of cables which need to go round the "elevation wrap" and a full 360° rotation should be quite feasible.

When the antenna is required to move rapidly through short distances the two designs are quite comparable at high elevation angles, but the slant-axis needs more rapid rotations at lower elevations, particularly below 20°; down to 4° the degradation is less than a factor of 4, getting rapidly worse below that. Most sources will be above this so that the performance should not be significantly degraded.

In addition to the geometrical aspects discussed here, the dynamical ones need to be considered. The higher rotation rates could be offset by lower moments of inertia resulting from better location of turning axes and avoidance of counterweights. Also since torques are applied differently the structural resonances which are applied may be different. Further studies are needed in this area.

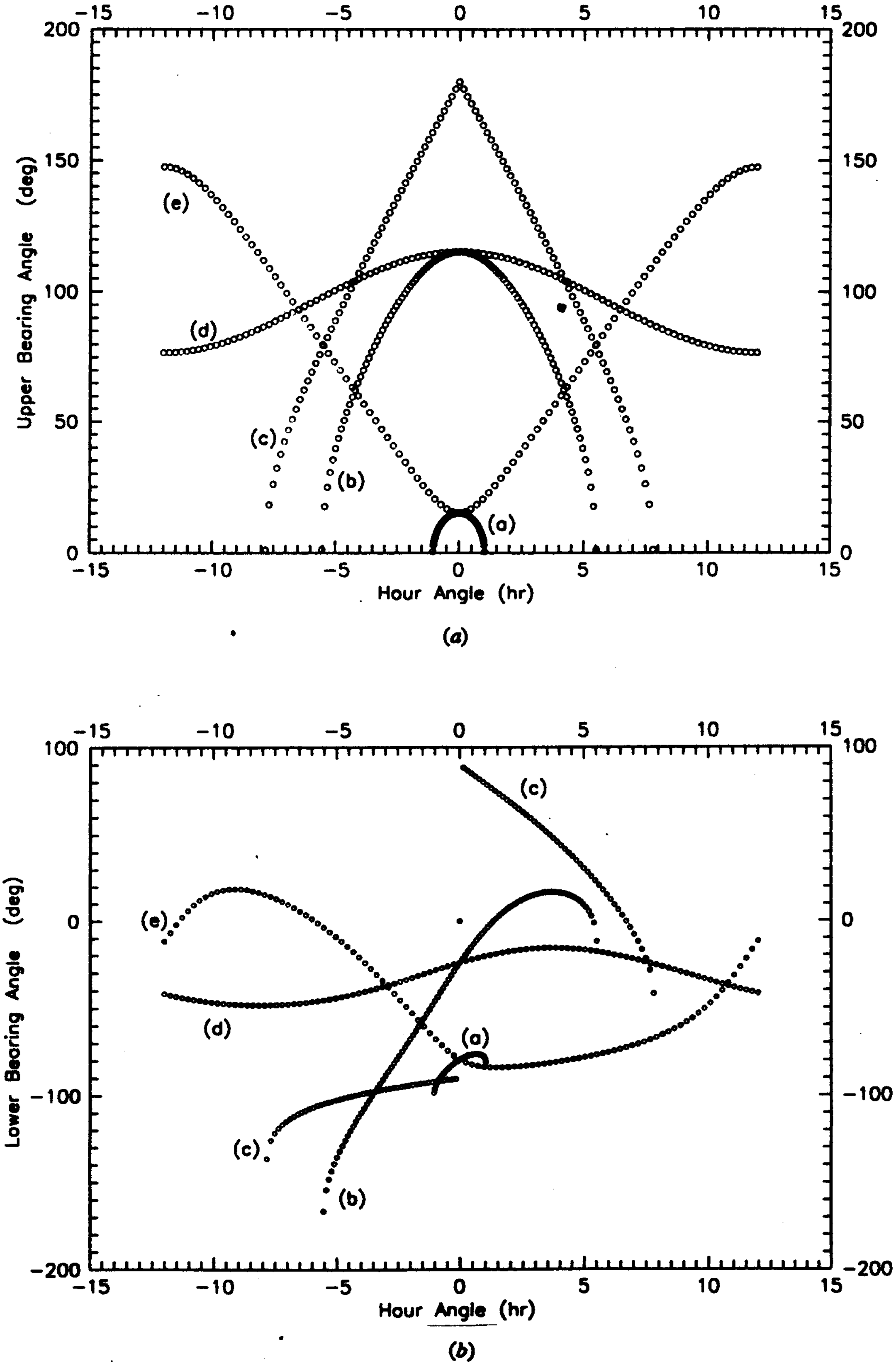


(a)



(b)

Figure 1. (a) Geometry of the slant-axis antenna.
(b) Coordinate definitions for pointing equations.



(a)

(b)

Figure 2. **Rotations about axes for tracking sidereal sources for slant-axis antenna.**

(*a*) "Elevation" axis
(*b*) "Azimuth" axis

Declination angles are (a) -54.9°, (b) -10.4°, (c) 34.1°, (d) 78.6°, (e) 123.1°.

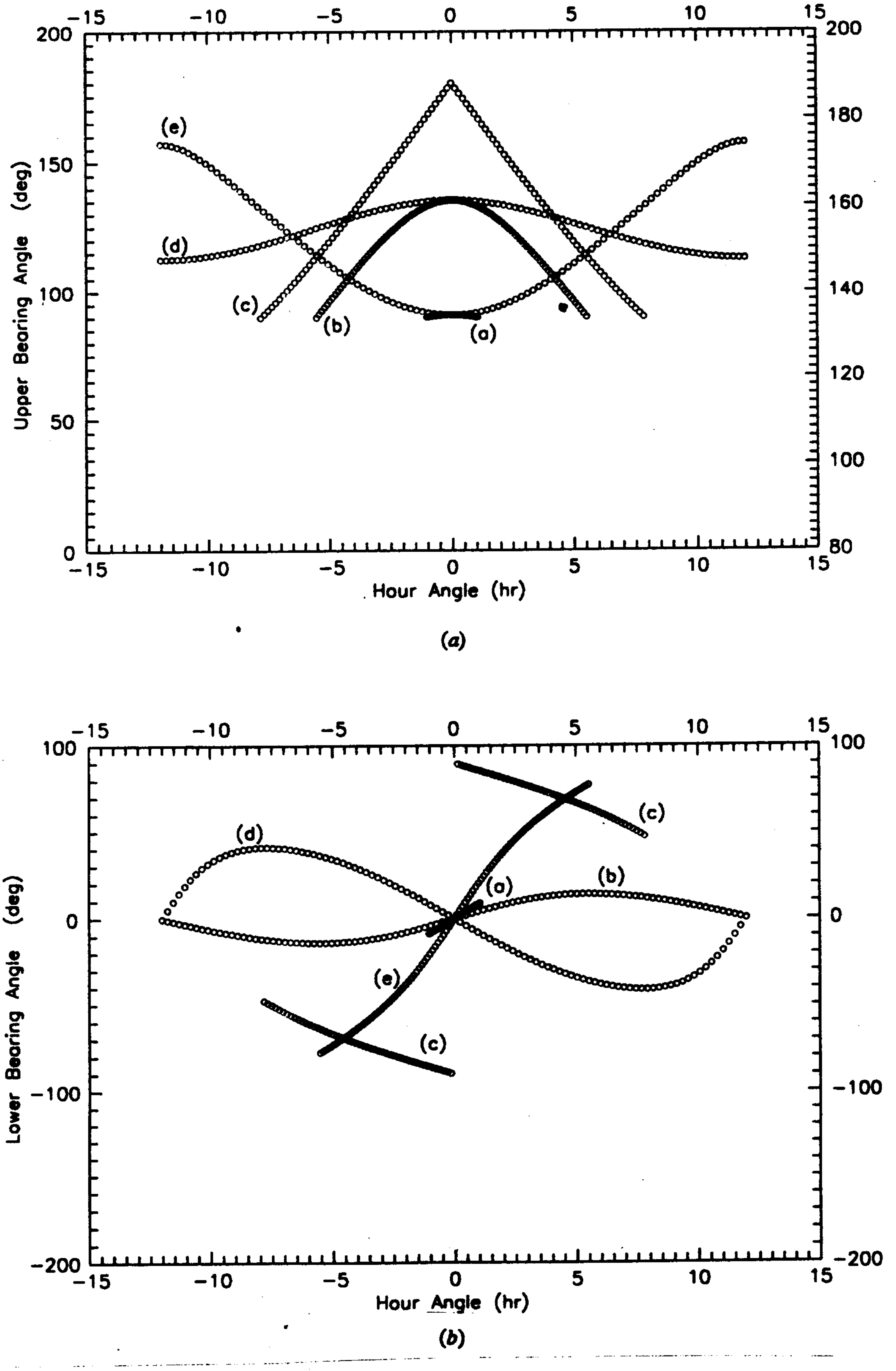


Figure 3. Rotations about axes for tracking sidereal sources for altazimuth antenna.

(*a*) "Elevation" axis
(*b*) "Azimuth" axis

Declination angles are (a) -54.9°, (b) -10.4°, (c) 34.1°, (d) 78.6°, (e) 123.1°.

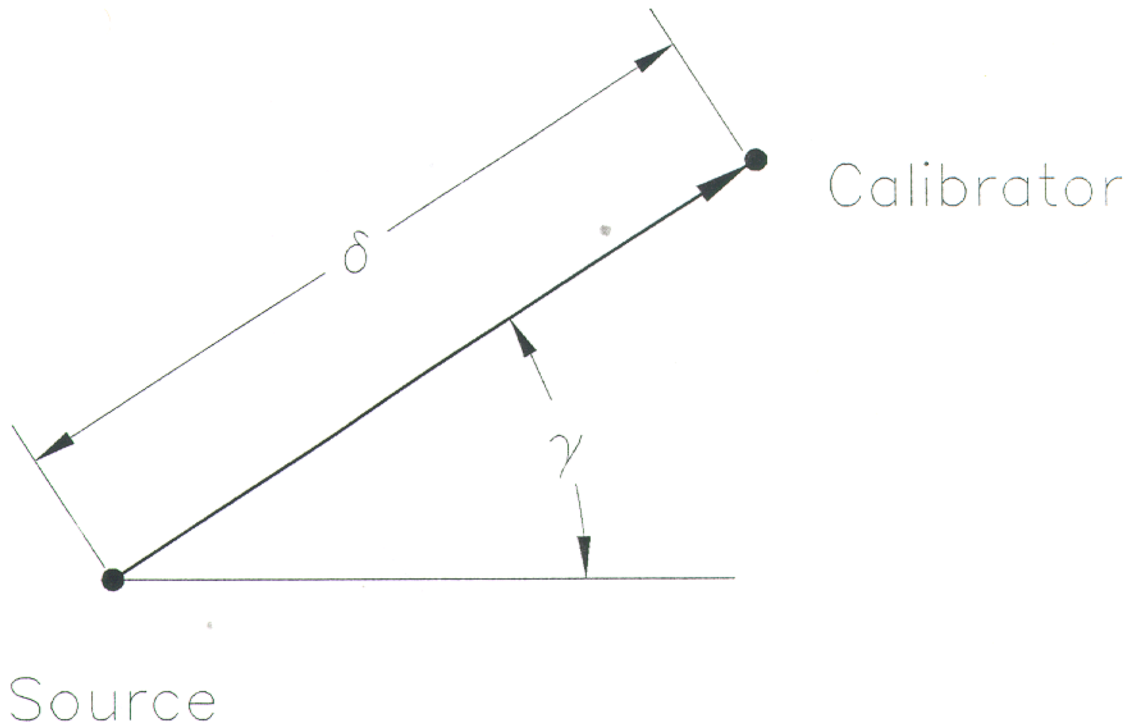
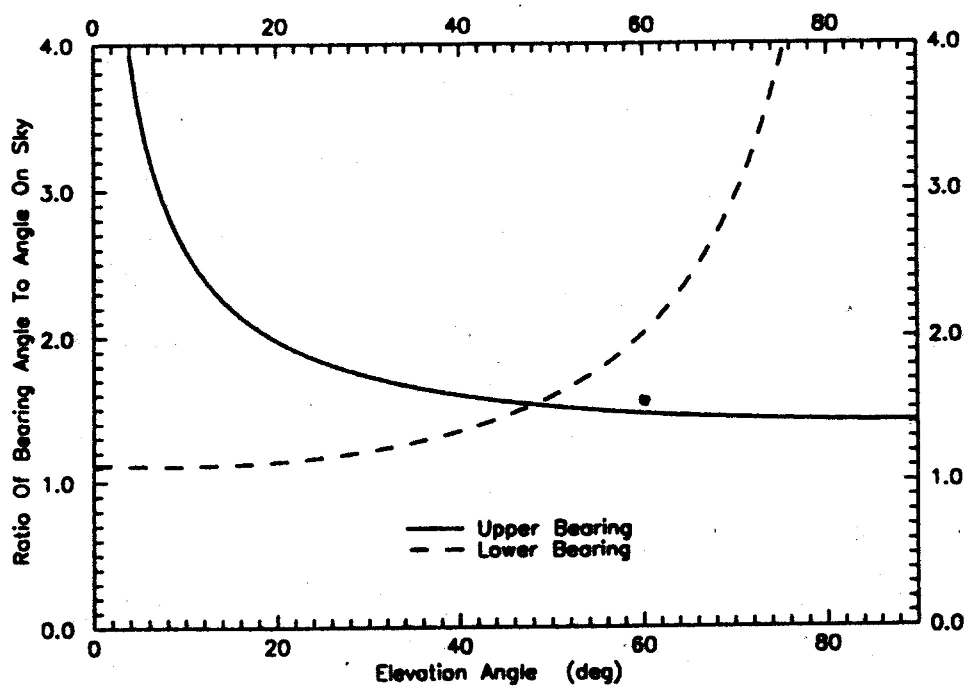
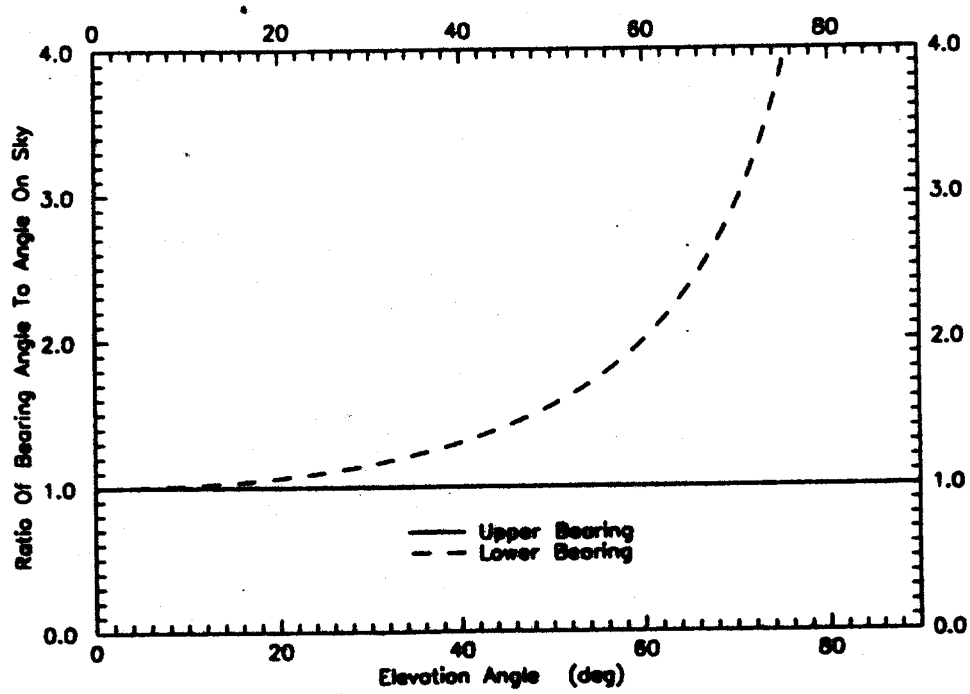


Figure 4. Geometry for source-calibrator switching.



(*a*)



(*b*)

Figure 5. **Maximum rotation of axes to switch to near-by calibration source.**

(*a*) **Slant-axis antenna**
(*b*) **Altazimuth antenna.**