

MEMORANDUM

**Date:** 12 July 1994  
**From:** James Lamb  
**To:** mmA Memo Series  
**Cc:**  
**Subject:** Erratum for Memo #13

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Dave Woody has pointed out an error in mmA Memo # <sup>98</sup>~~13~~, *Geometry Of The Slant-Axis Antenna*. The error is in equation (8) which has an incorrect sign. This affects also the text following and Figure 5(b). The correction introduces a singularity at  $0^\circ$  elevation for  $\alpha = \pi/4$ , in accord with what is physically expected.

Corrected versions of the relevant pages are attached.

#### 4 Small-Angle Switching Properties

One of the proposed observing modes for the array is to observe an astronomical source and to switch to a nearby calibration source at short time intervals to calibrate the atmospheric phase. We do not anticipate that the drive will be a problem for normal tracking, but this rapid switching between nearby sources on the sky will place a heavy demand on the servo system. Switching in azimuth will be similar for the conventional and slant-axis geometries, but movements with an elevation component will be different since both bearings of the slant-axis antenna need to turn. Consider a requirement for switching through some small angle  $\delta$  on the sky in a direction at an angle  $\gamma$  to the horizontal (Fig. 4). In terms of the change in elevation, and azimuth

$$\delta^2 = \Delta\theta^2 + \Delta\psi^2 \sec^2 \theta \quad (3)$$

and

$$\tan \gamma = \frac{\Delta\theta}{\Delta\psi \sec \theta} \quad (4)$$

The rotations required of the two bearings are related to the azimuth and elevation changes by

$$\Delta\theta' = \frac{\partial\theta'}{\partial\theta} \Delta\theta + \frac{\partial\theta'}{\partial\psi} \Delta\psi \quad (5)$$

and

$$\Delta\psi' = \frac{\partial\psi'}{\partial\psi} \Delta\psi + \frac{\partial\psi'}{\partial\theta} \Delta\theta \quad (6)$$

The partial derivatives may be found analytically and the following expressions give the bearing rotations in terms of  $\delta$  and  $\gamma$

$$\frac{\Delta\theta'}{\delta} = \frac{-\cos\theta}{\sqrt{\cos^2\theta + 2(\sin\theta - 1)\cos^2\alpha}} \sin\gamma \quad (7)$$

and

$$\frac{\Delta\psi'}{\delta} = \frac{\cos\alpha(\sin\theta - 1)}{\cos\theta\sqrt{\cos^2\theta + 2(\sin\theta - 1)\cos^2\alpha}} \sin\gamma - \frac{\cos\gamma}{\cos\theta} \quad (8)$$

Fig. 5a and Fig. 5b show how the required bearing rotations depend on the elevation of the source. For the conventional antenna (Fig. 5b) the maximum angle that the elevation bearing turns is the same as the angle between the source and the calibrator and occurs, of course, when  $\gamma = 90^\circ$ . The azimuth bearing rotation depends on the elevation of the source and large angles are needed when  $\gamma = 0^\circ$  at high elevations. The slant-axis antenna (Fig. 5a) behaves much like the slant-axis at high elevations. However, at low elevations, both bearings need to move further than  $\delta$ , and the increase in angle is similar to that required for the azimuth axis at high elevations. The worst values for  $\delta$  vary between  $0^\circ$  at  $90^\circ$  elevation and  $90^\circ$  at  $0^\circ$  elevation.

These results show that both axes in the slant-axis antenna need to be driven further than the angle between the source and the calibrator, but the worst cases occur at the opposite elevation extremes. Furthermore, the magnitude of the effect is no worse than for the conventional antenna, although that is affected in only one axis and

at one elevation extreme.

## 5 Discussion

The slant-axis antenna is more complicated in geometry than the conventional altazimuth one. Generally the axes of the slant-axis antenna need to be rotated further than those of the conventional design (though the moments of inertia may be smaller round those axes). Complete sky coverage is obtained if the slant bearing rotates between  $0^\circ$  and  $180^\circ$ , but if it is also permitted to rotate between  $180^\circ$  and  $360^\circ$  lower rotation rates for the azimuth bearing could be achieved for some parts of the sky. If the receivers are located below the slant axis there will be only a small number of cables which need to go round the "elevation wrap" and a full  $360^\circ$  rotation should be quite feasible.

When the antenna is required to move rapidly through short distances the two designs are quite comparable at high elevation angles, but the slant-axis needs more rapid rotations at lower elevations, particularly below  $20^\circ$ ; down to  $4^\circ$  the degradation is less than a factor of 4, getting rapidly worse below that. Most sources will be above this so that the performance should not be significantly degraded.

In addition to the geometrical aspects discussed here, the dynamical ones need to be considered. The higher rotation rates could be offset by lower moments of inertia resulting from better location of turning axes and avoidance of counterweights. Also since torques are applied differently the structural resonances which are applied may be different. Further studies are needed in this area.

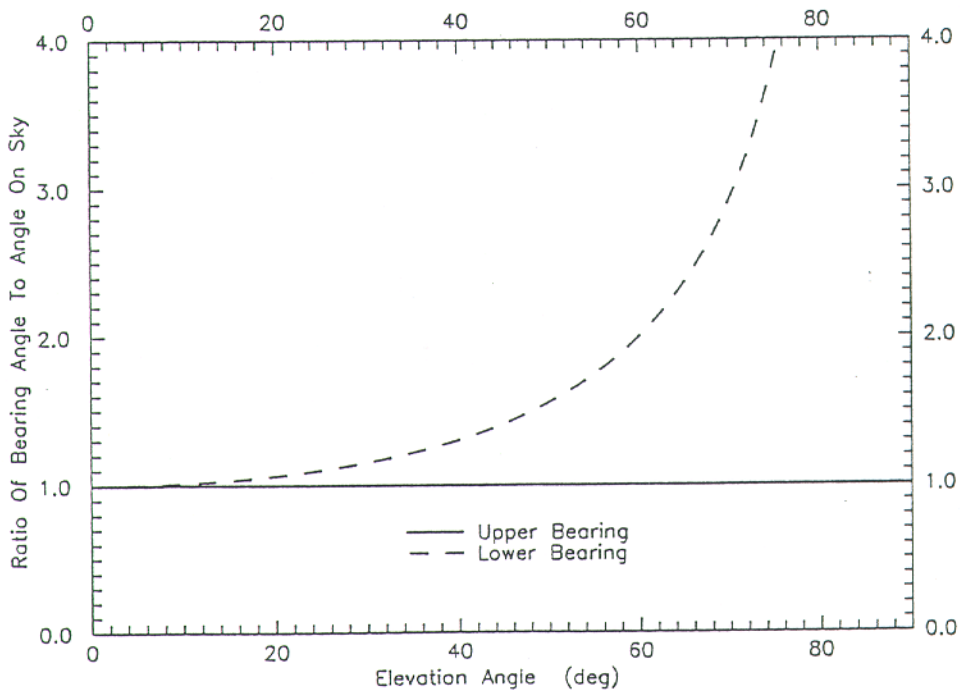
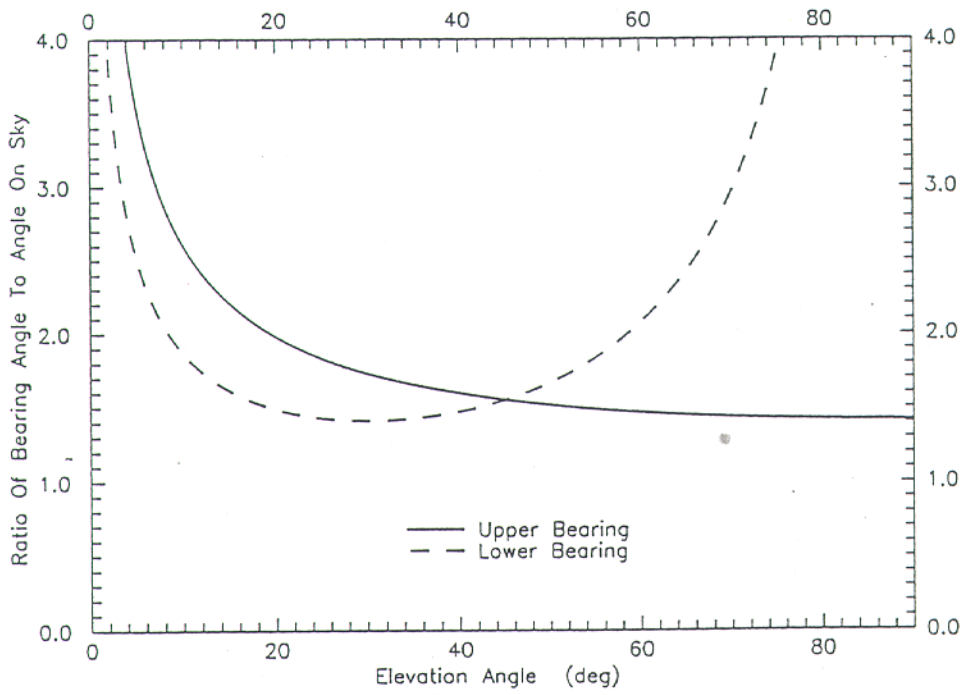


Figure 5. Maximum rotation of axes to switch to near-by calibration source.

- (a) Slant-axis antenna
- (b) Altazimuth antenna.