# Removal of Atmospheric Emission from Total Power Continuum Observations

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**Abstract**

We use the Chajnantor, Chile phase monitor data to determine the total power error made in subtracting the atmospheric emission for beam switched and On-The-Fly (OTF) continuum observations. When the error in atmospheric removal is less than the system noise per switching interval, the total power observations will not be limited by errors in atmospheric subtraction. If the errors in the atmospheric removal are larger than the system noise per switching interval, they will dominate the total power noise but should average down if a systematic atmospheric trend is removed by double beam switching or by OTF scanning.

On the Chajnantor site, switching on time scales of 1 s will usually increase the continuum total power noise over the optimum value by about 50%. If switching can be done on time scales of 0.2 s, the atmosphere will only rarely limit the noise. Using the "fast switching" capabilities which appear to be required for phase calibration, it should be possible to switch the primary by a few arcminutes on 1 s time scales. However, with slew speeds of $\sim 1$ degree per second, On-The-Fly (OTF) continuum single dish observations of sources up to a few degrees across will usually not be limited by atmospheric subtraction.

These calculations indicate that a nutating subreflector may not be required for the MMA if fast OTF mapping can be used for measuring the total power. Scanning faster than about 1 degree per second will not improve the atmospheric subtraction since the errors will then be dominated by the differing paths through the atmosphere at opposite ends of the extended target source. In order to accommodate 1 degree per second scanning, the correlator will need to record integration times as short as 0.003 s, which may be possible for total power data alone, but will likely not be possible for the visibility data. This will require that the total power and interferometric data be taken separately for many continuum observations.

## 1 Introduction

Water vapor in the atmosphere, especially inhomogeneously distributed water vapor, is a bad thing for millimeter wavelength telescopes. Single dish observations are mainly affected by the opacity of the water vapor and by the variable emission from the inhomogeneously distributed water vapor. The problem of variable atmospheric emission has been solved to a large extent by the dual beam, or beam switching, observational technique of Emerson, Klein, and Haslam (1979). Two beams are formed on the sky, either by observing with two physical feeds simultaneously or by switching the sky position of the beam by chopping the secondary reflector or by actually changing the pointing of the telescope. It is assumed that the atmospheric emission in the "off" beam and the "on" beam are similar, so when the "off" power is subtracted from the "on" power, only the astronomical emission remains. Success of the atmospheric emission subtraction depends on how similar the atmosphere really is in the "off" and "on" beams, which depends upon how close the "off" and "on" columns are to each other in the atmosphere: beam switching works best when small angular beam throws and high switching frequencies are used. However, there has not been a good analysis of the switching speed and angular distance requirements for total power observations, primarily because the atmospheric fluctuations have not been well characterized at the sites of interest until recently.

For extended sources, both the "on" and "off" beams will fall on the source of interest, so some astronomical emission is subtracted as well as the atmospheric emission. Various deconvolution algorithms can reconstruct the image using the information from the "on" and "off" beams (see Emerson, 1995 for a review). Because we must difference many "on" regions to get to the edge of the source, the signal to noise of a large reconstruction falls off like $\sqrt{N_{throw}}$, where $N_{throw}$ is the number of beam throws across the target source. Imaging very large regions of the sky in single dish continuum at high sensitivity then becomes very problematic.

Recently, David Woody has suggested that the MMA antennas might not require a nutating subreflector if the MMA were built on a superior site and the antennas were able to position switch a few arcminutes at ~1 Hz. If the atmosphere is stable enough, On-The-Fly continuum observing may be able to produce good images of very large regions without the signal to noise degradation which the beam switching technique suffers from. This memo performs an error analysis of the beam switching and OTF single dish continuum observing techniques using the atmospheric stability data from our Chilean site test interferometer. We compare the observing techniques at a frequency of 230 GHz and address the question of whether a nutating subreflector is required for the MMA.

## 2 The Form of the Error Analysis

The atmospheric emission as seen by a single dish will depend upon the opacity and the temperature of the atmosphere:

$$T_B = T_{sky}(1 - e^{-\tau}) \simeq T_{sky}\tau. \tag{1}$$

The rate of evolution of the atmosphere will be slow compared to the times of interest pertinent to the beam switching problem, so we adopt the frozen turbulence model in which all temporal fluctuations are assumed to result from spatial fluctuations blowing past the dish.

## 2.1 Are Sky Temperature Fluctuations Important?

Spatial fluctuations in either the opacity $\tau$ or sky temperature $T_{sky}$ will cause fluctuations in the atmospheric emission received by the single dish. We will find later on in this work that the errors in the sky emission subtraction due to temporal and spatial fluctuations of $\tau$ will be of the order of 0.01 K. Will the temperature microstructure of the atmosphere be able to contribute a comparable error? If so, then these calculations are a lower limit to the atmospheric errors in continuum single dish observing. If the turbulent water vapor is distributed uniformly through a $\sim$1 km atmosphere, isotropic temperature variations of 2 K over 10 m and an opacity of $\tau = 0.05$ will result in variations in the sky brightness of about 0.01 K without any opacity fluctuations. During some conditions, it appears that a narrow layer is responsible for the phase fluctuations seen by the site test interferometer as the phase structure function exponent is seen to be about 0.33. This just requires that most of the water vapor *fluctuations* reside in a thin layer. However, if a large fraction of the *water vapor* also is in a thin layer, even modest temperature fluctuations will be enough to dominate the sky brightness fluctuations. Measurements of the atmospheric temperature structure function near the earth's surface indicate that 0.5 K over 10 m is not uncommon (Tatarski, 1961). If the temperature microstructure decreases with height in the atmosphere, temperature fluctuations will probably not dominate the fluctuations in the sky brightness.

This is one of the issues involved in correcting for atmospheric phase errors using water vapor radiometry. Foster *et al.* (1995, in preparation) demonstrate that sky brightness fluctuations measured by the 225 GHz site test radiometer imply rms path length fluctuations which agree with the phase fluctuations measured by the 11.7 GHz site test interferometer on the Chajnantor site. This correlation between brightness fluctuations and path length fluctuations indicates that we are justified in scaling the phase fluctuations to estimate the magnitude of the sky emission fluctuations as seen by a single dish.

## 2.2 Scaling Between Path Length and Sky Brightness Fluctuations

We assume the microstructure of the opacity dominates the sky brightness fluctuations and the microstructure of the physical temperature of the sky is insignificant. Then the errors made in the atmospheric emission subtraction are similar to the errors made in fast switching phase calibration:

$$\Delta T_B \simeq T_{sky} \sqrt{D_\tau(|\mathbf{v}t/2 + \mathbf{d}|)}, \tag{2}$$

where $D_\tau$ is the *opacity* structure function, $v$ is the atmospheric velocity, $t$ is the switching cycle time, and $d$ is the typical distance between the centers of the "on" and "off" near field columns. To use Equation 2 requires a model of the height of the turbulent water vapor to determine $d$. If the turbulent water vapor is primarily a surface phenomenon, then $d$ is essentially zero. For water vapor at 1 km height above the telescope and a 1 arcminute beam throw, $d = 0.29/\sin(el)$ meters.

If the distance between the "on" and "off" lines of site is much larger than the distance the atmosphere moves in half a cycle time, as for very fast switching with a large beam throw or for fast OTF mapping of a large source, Equation 2 approaches

$$\Delta T_B \simeq T_{sky}\sqrt{D_\tau(d)}. \tag{3}$$

If the distance the atmosphere moves in half a cycle time is much larger that the distance between the "on" and "off" lines of site, as for slow switching with a small beam throw, Equation 2 approaches

$$\Delta T_B \simeq T_{sky}\sqrt{D_\tau(vt/2)}. \tag{4}$$

We have techniques to estimate $v$ from the site test data, but we can bypass the need for knowledge of $v$ entirely by expressing 4 in terms of the temporal structure function $\bar{D}_\tau(t)$, which can be derived from the phase time series:

$$\Delta T_B \simeq T_{sky}\sqrt{\bar{D}_\tau(t/2)}. \tag{5}$$

Under the assumptions that the opacity fluctuations and the differential path length fluctuations are solely due to water vapor, the opacity structure function is simply related to the phase structure function. The opacity fluctuations are given by

$$\Delta\tau = a(\nu)\Delta PWV, \tag{6}$$

where $PWV$ is the precipitable water vapor in millimeters and $a(\nu)$ is assumed constant over the observed bandpass. For 230 GHz, $a \simeq 0.03$ at the elevation of the Chajnantor, Chile site (Schwab, private communication)[1] If the path length fluctuations seen by our site test interferometer are caused solely by inhomogeneously distributed water vapor,

$$\Delta l = 6.3\Delta PWV \tag{7}$$

(Thompson, Moran, and Swenson, 1986). Then the opacity structure function is related to the path length structure function, which is measured by the site test interferometer, as

$$\sqrt{D_\tau(r)} = a(\nu)/6.3\sqrt{D_l(r)}. \tag{8}$$

Inserting this relationship into Equations 2 through 5, we can use the site test data to estimate the errors in atmospheric emission cancellation for different observing strategies. In order to convert the atmospheric noise from Kelvins to Janskies for the MMA antennas in total power, we multiply by 46.

[1] Fred Schwab has run Liebe's atmospheric transmission code on water vapor profiles taken from Tucson and Mauna Kea radiosonde launches, comparing the modeled opacity to the integrated PWV above some height. Schwab's best relationship between opacity and PWV is dependent upon the minimum elevation cutoff, and for the 5000 m Chajnantor, Chile site, $\tau = 0.0089 + 0.0031PWV + 0.0015PWV^2$. We have dropped the quadratic term to simplify our analysis.

### 2.3 Does the Site Test Interferometer Measure Fluctuations on the Right Time Scales?

The site test interferometer measures the rms path length fluctuations over a 300 m baseline, but the total power problem requires knowledge of fluctuations on baselines of 1-10 m. We produce a temporal structure function from the interferometer's phase time series (Holdaway *et al.* 1995), which provides information about the path length variations on time scales down to 1 s. Our site test data shows no evidence for deviations from a constant power law down to 1 s. However, at 1 s we are often affected by the interferometer's instrumental function, and it is not clear if we could see a change in the structure function. We will assume that the structure function which we derive on time scales between 1 s and the turnover time, typically 20-60 s, is representative of the structure function on time scales of 0.1 s to a few seconds.

## 3 Observing Strategies

We are now prepared to analyze these observing modes:

- **Spectral line observing.** In spectral line observations, each channel will have a very narrow bandwidth, so much larger errors in the atmospheric emission cancellation can be tolerated before the spectra are limited by the atmosphere. This implies that much slower switching can be tolerated. In addition, if the spectrum of the atmospheric emission in the bandpass does not change with time, a baseline can be fit to the observed spectrum and subtracted. Because of these factors, spectral line single dish observing will not be the most demanding case and will not be considered further.

- **Beam switched single pointing continuum observations.** Continuum observing will be the most demanding case for subtracting the atmospheric emission. Routinely, single dishes observe in continuum with a nutating subreflector subtracting the signal in the "off" beam from the signal in the "on" beam. However, an advancing wedge of water vapor will result in systematic errors between the "on" and "off" beams which will not average down with time. To guard against this situation, *double beam switching* is employed, using two "off" positions placed symmetrically about the "on" position. We will assume that if double beam switching is used, there will be no systematics due to the atmosphere and that any residual subtraction errors will average down.

  Since the beam throw distance will be limited by the optics in beam switching, it will take multiple throws to reach the zero-emission level when imaging a large source, increasing the noise by $\sqrt{N_{throw}}$. This becomes quite problematic for very large sources.

- **Position switched continuum observations.** At lower frequencies, it is not uncommon to observe with position switching, driving the entire telescope between the "on" and "off" positions. Position switching will allow arbitrarily large beam throws, but at the expense of large cycle times and large distances between the lines of sight $\mathbf{d}$. Since position switching does not require any off-axis observing, the beam shapes for the "on" and "off" will be more nearly identical than in the beam switching case. The error analysis for position switching is identical to beam switching, but with lower switching frequencies.

- **On-The-Fly (OTF) continuum observations.** Quickly scanning over the source of interest, or OTF mapping, will also enable subtraction of the atmospheric emission if the pointings at both ends of the scan are free of astronomical emission. Because there is an "off" at both ends of the scan, subtracting a baseline from the scan is similar to double beam switching as it removes any systematic errors. Stopping and accelerating the telescope will actually require a fair bit of time, resulting in more time spent on the "off" band about the astronomical source to be greater than time spent on any "on" position. This will reduce the system noise in the "off" measurements, thereby improving the system noise in the "on" minus "off". However, if errors in atmospheric emission subtraction dominate the system noise, this increase in SNR will not be realized.

### 3.1 Error Analysis for Beam Switched or Position Switched Continuum Observations

At 230 GHz, the MMA dishes will have a 40 arcsecond beam, and the beam throw might be 80 arcseconds. At 1 km height above the MMA, the centers of the "on" and "off" near field columns will be about 0.4 m when observing overhead. Hence, with wind speeds of 10 m/s, $vt/2 \gg d$ for cycle times of of 0.4 s or greater. By ignoring the spatial term, we can perform our error analysis in terms of the temporal structure function and completely ignore the issue of wind velocity. Using the temporal structure functions derived from the June 1995 site test data from Chajnantor near San Pedro in Chile and Equation 5, we can derive the distribution of the error in atmospheric cancellation for different switching frequencies. Figure 1 illustrates the distributions of atmospheric cancellation error for beam switching at 0.5, 1, 2, and 5 Hz. At 5 Hz, the spatial term will be comparable to the temporal, so the errors will likely be somewhat larger than the graph portrays. As the switching frequency increases, we decrease the atmospheric cancellation error and increase the system noise per cycle time[2], both working against limitation by the atmosphere. The atmosphere will limit the continuum total power sensitivity about half the time when switching at 2 Hz, and much less than half the time when switching at 5 Hz.

David Woody has suggested that *position switching* by several arcminutes at 1 Hz should be possible with the MMA antennas, and that this should be considered as an alternative to beam switching for single dish continuum observations. While Figure 1 indicates the MMA in single dish mode will not often reach its ultimate sensitivity with 1 Hz switching, it will often be close to its ultimate sensitivity. Hence, we should seriously consider position switching as a means for the MMA to remove atmospheric emission in continuum total power observations, eliminating

[2] For total power observations switched against blank sky with half the time on source and half the time off source, the noise goes like $2kT_{sys}/sqrt\Delta\nu\Delta t$, losing one factor of $\sqrt{2}$ by observing the source only half the time and another factor of $\sqrt{2}$ by differencing two noisy signals.

the requirement for a nutating subreflector. Note that position switching will almost never limit the sensitivity for spectral line observations.

### 3.2 Error Analysis for OTF Continuum Observations

Since On-The-Fly mapping generally deals with large sources, certainly much larger than the ~1 arcminute beam throw suggested in the previous section, we must consider spatial variations in the atmosphere's brightness temperature, but if the telescope slews fast enough, we don't need to consider temporal fluctuations caused by the atmosphere moving while we observed. This allows us to use Equation 3 to calculate the errors in cancellation of the atmospheric emission, which will depend only on source size. For the case of OTF mapping, the distance $d$ is given by the half the distance between the lines of sight to opposite ends of the target source, at the mean height of the atmospheric turbulence. It doesn't make sense to slew so fast that $vt/2 \ll d$. Equating the temporal and the spatial terms, we find the slew rate at which the two terms contribute equally:

$$v_{slew} \simeq v_{atmos} 180/(\pi) \sin(\theta_{el})/h \tag{9}$$

degrees per second, where $h$ is the typical height of the atmosphere and $\theta_{el}$ is the elevation angle. Hence, a slew rate of one degree per second for 1000 m water vapor scale height and 10 m/s wind velocity will be fast enough so that the atmospheric cancellation error is dominated by the distance between the lines of site to the two different ends of the target source, rather than by the temporal variation in the atmosphere over the observations.

Figure 2 indicates the distribution of error in atmospheric cancellation for four different source sizes. Assuming a slew speed of 1 degree/s for all sources, the integration time per Nyquist sample will be 0.005 seconds, implying a system noise per Nyquist sample of about 2.0 Jy, considering that the system noise in the "off" signal will not increase the noise by $\sqrt{2}$ because of its higher SNR due to longer integration on the "off" position during telescope reacceleration. Hence, very fast OTF mapping will seldom be dominated by errors in atmospheric cancellation untill observed sources are much larger than a degree. OTF mapping will perform significantly better than beam switching or position switching for small sources, though more time will be wasted off source. For sources smaller than a degree across, a much slower scan rate will still result in OTF observations which are not limited by the atmosphere.

These results have a large impact on several areas of the MMA:

- **continuum observations of very large sources** are now possible: OTF mapping will have $\sim 7\sqrt{2}$ better SNR on a one degree source than traditional beam switching (49 beam throws across the image degrades the traditional beam switching image SNR by 7, and differencing from a very low noise off in OTF mapping improves the OTF image SNR by $\sqrt{2}$).

- **correlator dump time:** slewing at one degree per second and recording data four times per beam (so the beam motion per integration is much smaller than the beam) requires that the correlator be able to dump the data every 0.003 s. Since spectral line total power observations are not so sensitive to the atmosphere, this fast data dump rate is only required for continuum observing, and continuum data will only contain a few channels per antenna per integration time. Furthermore, there are no atmospheric reasons which require such fast correlator dump times for the continuum interferometric visibilities. Under this scenario, OTF continuum total power observations would be taken separately from the continuum interferometric measurements, but spectral line total power and interferometric data would still be taken simultaneously, but not in such a fast OTF mode.

- **antenna design:** it is now possible that the antennas do not require a nutating subreflector, thereby simplifying the antenna design and saving the MMA project many dollars.

## 4 Potential Problems With These Methods

These methods represent a major change in the way total power continuum measurements would be made. In order to test these ideas fully, we would need to build the MMA, as no other instrument is capable of performing these observations as quickly. Some atmospheric testing at slower slew rates and with conventional beam switching with existing telescopes, in concert with phase monitor observations, will undoubtedly add to our understanding. In addition to bolstering the basic theory, we will have to address a number of technical concerns:

- How accurately will the antenna encoder positions be under very high accelerations which will occur to get the antennas up to 1 degree/s? This is new technical territory which need to be explored.
- Can the correlator dump $\sim 40 \cdot 8$ numbers every 0.003 s?
- The fast OTF mode will be the method of choice since it will work well even with modest atmospheric stability. However, the OTF mode will not be very efficient for smallish (ie, a few single dish beam widths) sources. The position switching observing technique may be more efficient than OTF for smallish sources, but the switching is probably slower, and will be at least marginally limited by the atmosphere.

## Acknowledgements

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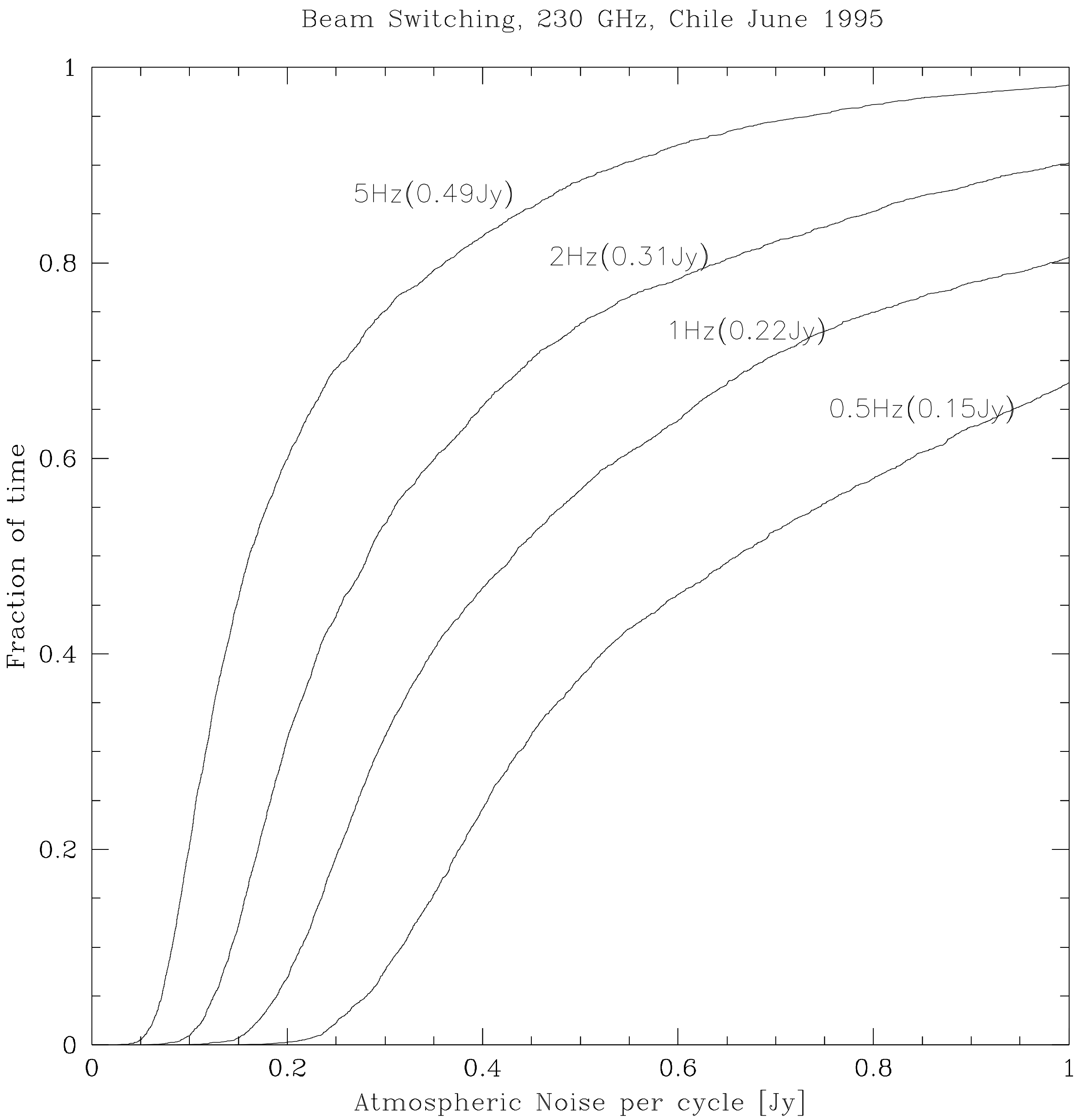


Figure 1: Distribution of the atmospheric emission cancellation error for beam switched continuum observations at 230 GHz for four different switching frequencies, assuming the temporal variations dominate the spatial variations. At 5 Hz, the spatial and temporal fluctuations will be comparable, so this approximation breaks down. Also listed on each curve is the thermal noise obtained from 4 GHz total bandwidth (two stokes from two 1 GHz IFs) per half cycle time. As the switching frequency increases, the cancellation errors decrease and the system noise per half cycle increases. In order for the atmosphere not to limit the sensitivity, the atmospheric errors must be less than the system noise.

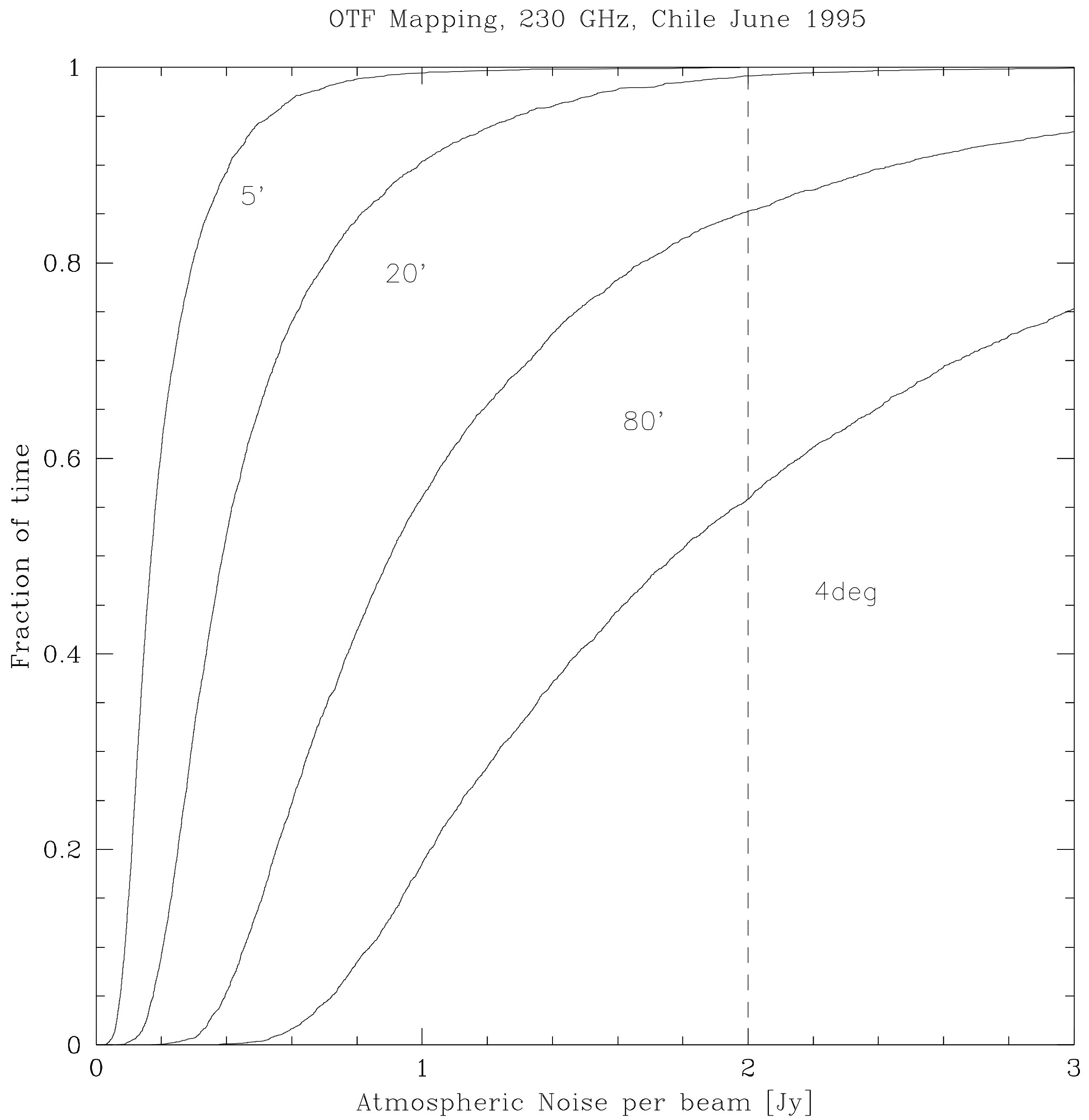


Figure 2: Distribution of the atmospheric emission cancellation error for On-The-Fly single dish continuum mapping at 230 GHz for four different source sizes, assuming the spatial variations dominate the temporal variations, as is met of the slewing speed is greater than about 1 degrees/s. At a slew rate of 1 degree/s, the noise per beam is about 1.4 Jy, so even sources as large as 4 degrees can often be mapped without being limited by the atmosphere. Smaller sources will not require such fast slew rates and short integrations.