MMA Memo 171: Optimization of an Array Configuration Minimizing Side Lobes

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Abstract

The analytical expression for the first derivative of the array beam by the specific array element shift is obtained. Using this expression it is possible to optimize the array configuration shifting the array elements in accordance of the derivative value. The criterion of the optimization can be selected as minimization of the worse side lobe at the each selected cross section as well as minimization of the worst side lobe at the whole selected range of the angles. A task in AIPS frame is written to apply the algorithm for the optimization. The task provides the optimisation of the array's element position and plots the array's pattern for initial and found configuration as well as the configurations itself. We consider the main result of this investigation is a demonstration of the tool which can be used for optimization of the side lobes of designed arrays. Even if the recommended configuration is not accepted for different reasons it is useful to know how far the side lobes of the accepted configuration are from the "ideal one". The offered analysis can answer on this question.

1 The approach to the problem

Let's suppose the vector $\vec{r}_i$ determines the position of the array element at the aperture of the array measured at wavelengths. Then the beam pattern of the array is determined by all available baselines and can be verified by the following equation

$$P(\vec{e}) = \frac{1}{N^2}\sum_{k=1}^{N}\sum_{n=1}^{N} e^{-i2\pi(\vec{r}_k-\vec{r}_n)\cdot\vec{e}} = \frac{1}{N}\sum_{k=1}^{N} e^{-i2\pi\vec{r}_k\cdot\vec{e}} \cdot \frac{1}{N}\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}} = |U(\vec{e})|^2 \tag{1}$$

where $\vec{e}$ is the vector of the direction on the sky
$U(\vec{e}) = \frac{1}{N}\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}} =$ is a voltage beam pattern
N is the number of elements in the array.

Evaluating the derivative of (1) we can obtain the following expression of the beam differential depending on a shift of the given element:

$$dP_{\vec{r}_n}(\vec{e}) = \frac{4\pi(\vec{e}\cdot\Delta\vec{r}_n)}{N^2}\sum_{k=1}^{N}\sin 2\pi\left(\vec{r}_k - \vec{r}_n\right)\cdot\vec{e} \tag{2}$$

where $dP_{\vec{r}_n}(\vec{e})$ is the beam value change at the direction $\vec{e}$
if $\vec{r}_n$ is changed by $\Delta\vec{r}_n$

Combining equations (1) and (2) we can find the expression of relative change of the beam pattern depending on a shift of the given element:

$$\frac{dP_{\vec{r}_n}(\vec{e})}{P} = 4\pi(\vec{e}\cdot\Delta\vec{r}_n)\frac{\sum_{k=1}^{N}\sin 2\pi\,(\vec{r}_k-\vec{r}_n)\cdot\vec{e}}{\left|\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}}\right|^2} \tag{3}$$

2 Examples of application

I wrote a task in AIPS frame which provides the optimization of the side lobes for the given number of the cross section and the given range of the angles relatively of the perpendicular direction to the array aperture. Two algorithm of optimization are available: minimization of the worse side lobe at each selected cross section and minimization of the worst side lobe at the whole range of the angles. Ten cross sections at the the range $(0-10^\circ)$ with the step 18° have been selected. The partern of the array was estimated for the each cross section at the range of the angles from zero to twenty main lobes. The task calculates the differential of the array respond at the given direction for each element of the array and found the element correction following the formula:

$$\begin{aligned} dx(n) &= -G\cdot dP_{\vec{r}_n}\cdot cos(\alpha) \\ dy(n) &= -G\cdot dP_{\vec{r}_n}\cdot sin(\alpha) \end{aligned} \tag{4}$$

where α is the angle of the cross section of the array pattern
G is the gain at the iteration loop.

The correction for each array element is calculated interactively. At each iteration the position of the worse side lobe at each direction is estimated and new correction of the array elements is applied. The value of the gain in the loop is being found empirically although it is clear that the gain has to be less than l. I used gain at the range 0.0001 - 0.01. The task provides the plots of the initial and optimized array patterns and its configurations. I have applied the algorithm to circle arrays with gomogenios distribution of the elements along the circumference. The number of the array's elements varied from 12 to 40. The result is illustrated by figures (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15). The new found arrays have several times lower side lobes at the whole range of selected dirrections. The new found configurations are very different from initial circular one as long as the number of elements increases. Figures (16, 17, 18) demonstrate the algorithm application to the 40 elements array offered by M. Holdaway for Mouna Kea ([2]). Figures (19, 20, 21) demonstrate the algorithm application to the 12 elements array offered by T. Cornwell ([1]). In spite of the fact that the initial 12 element array is optimal by criteria of maximum distance between the U, V points ([1]) the side lobes are not optimal. The new found array has at least two times lower side lobes.

3 Conclusion

The offered method works only for snapshot type of observations. The synthesis using the Earth rotation can bring a difference. So the found configurations have to be checked at the full aperture synthesis scheme at the larger range of the angles. We consider the main result of this investigation is a demonstration of the tool which can be used for optimization of the side lobes of designed arrays. Even if the recommended configuration is not accepted for different reasons it is useful to know how far the side lobes of the accepted configuration are from the "ideal one". The offered analysis can answer on this question.

References

[1] T.J. Cornwell, MMA memo 38, 1986

[2] M.A. Holdaway, MMA memo 111, 1994

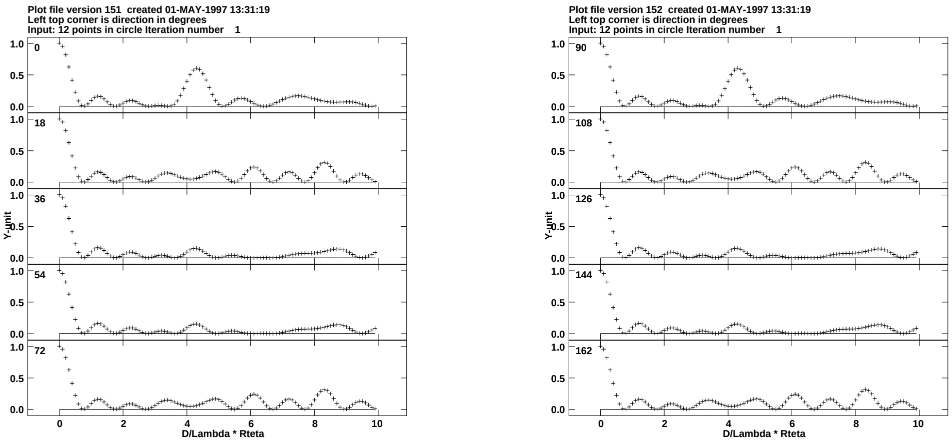

Figure 1: Beams pattern of the initial 12 elements array configuration

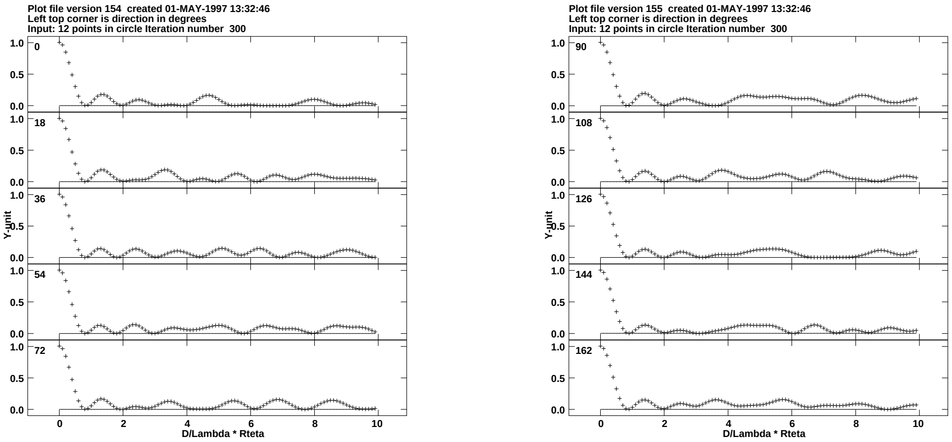

Figure 2: Beams pattern of the final 12 elements array configuration

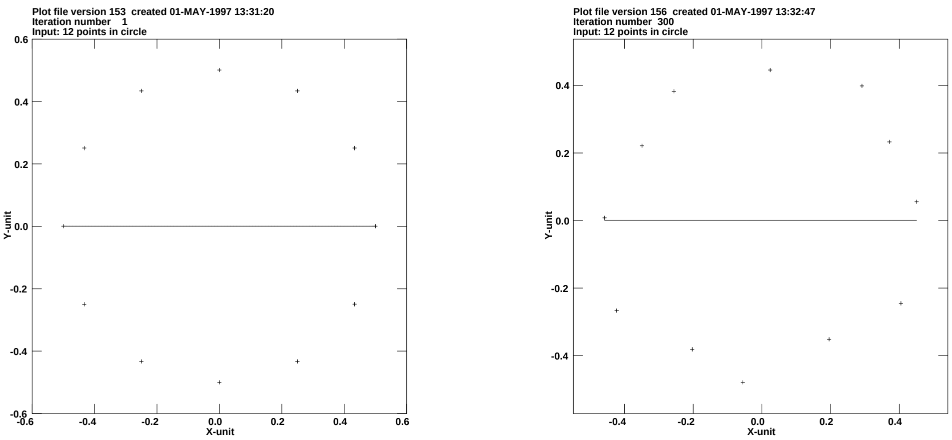

Figure 3: Initial and final configuration of the 12 element array

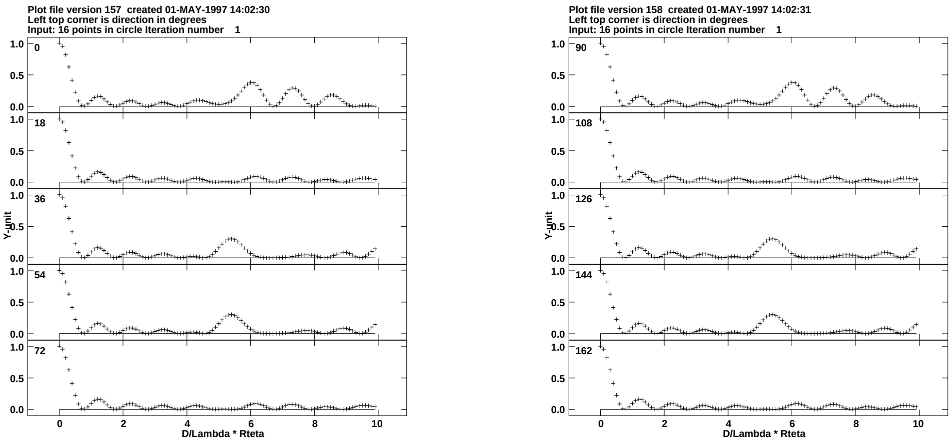

Figure 4: Beams partern of the initial 16 elements array configuration

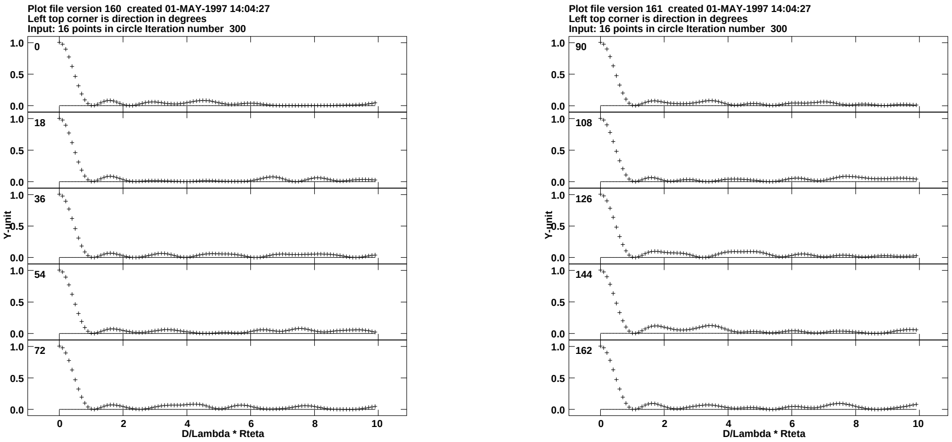

Figure 5: Beams pattern of the final 16 elements array configuration

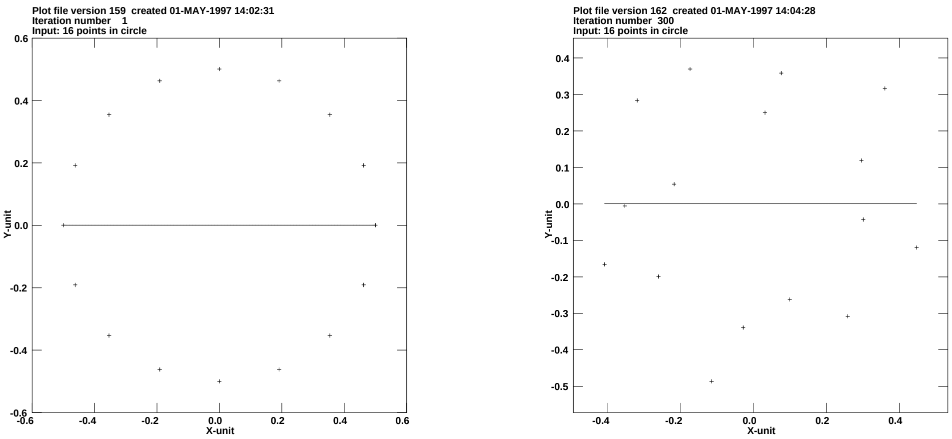

Figure 6: Initial and final configuration of the 16 element array

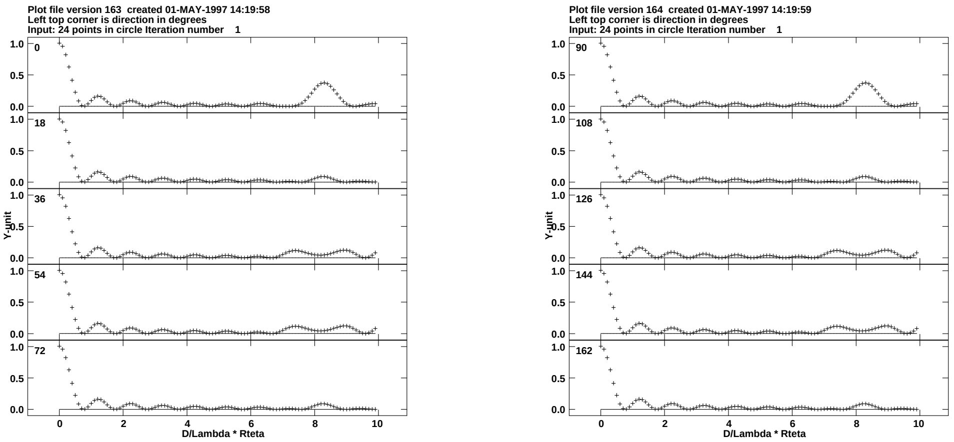

Figure 7: Beams pattern of the initial 24 elements array configuration

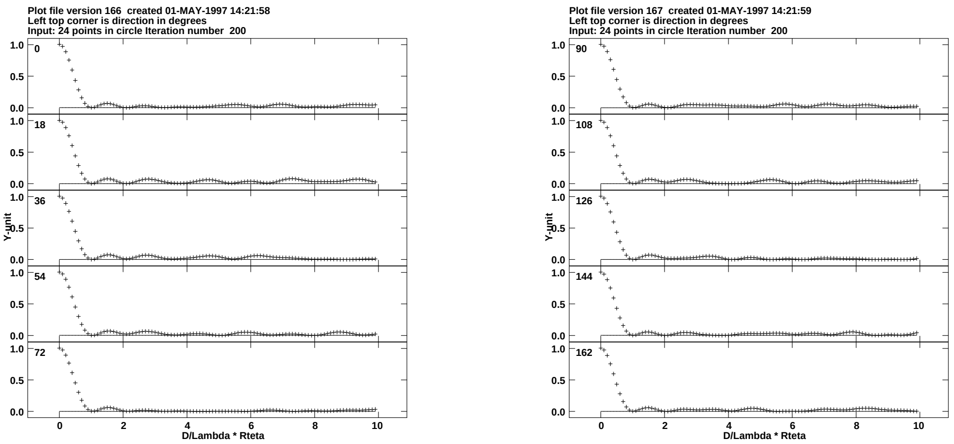

Figure 8: Beams partern of the final 24 elements array configuration

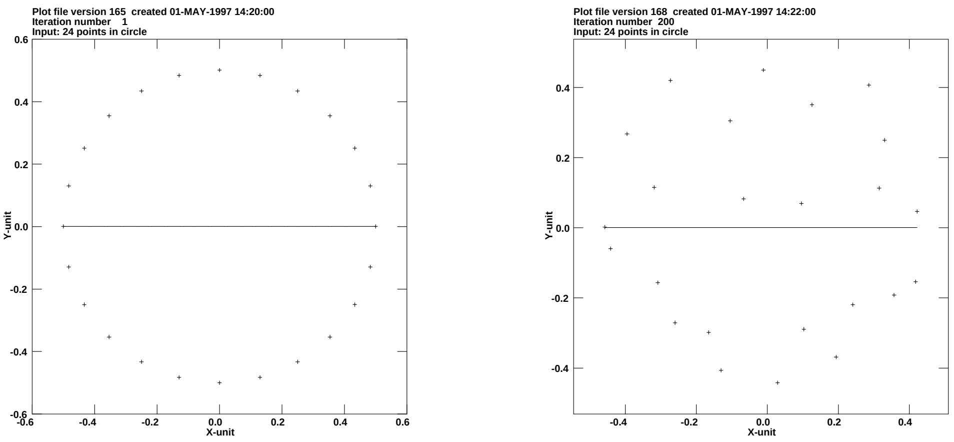

Figure 9: Initial and final configuration of the 24 element array

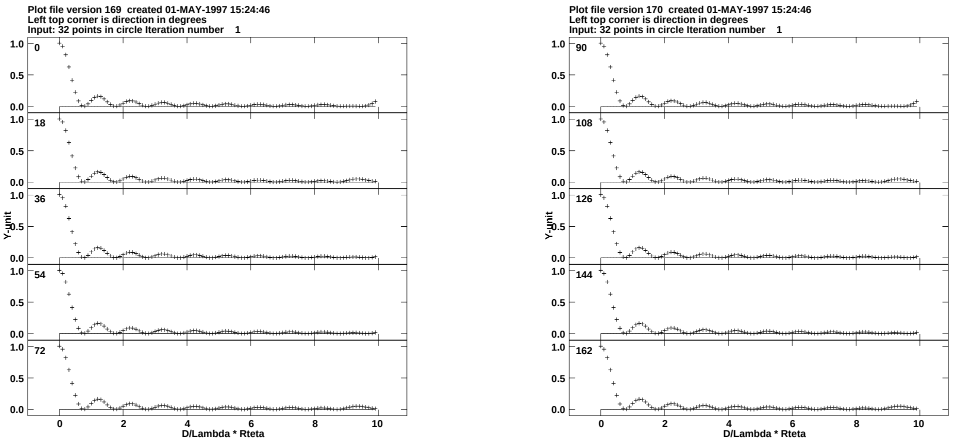

Figure 10: Beams pattern of the initial 32 elements array configuration

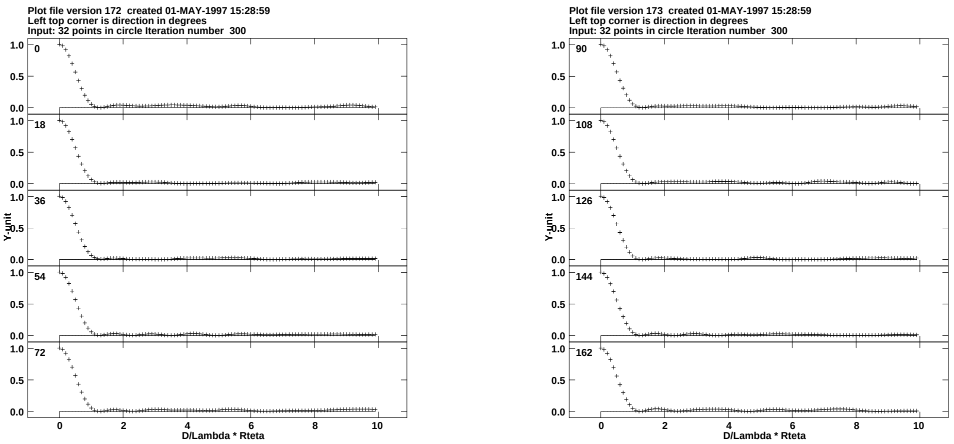

Figure 11: Beams pattern of the final 32 elements array configuration

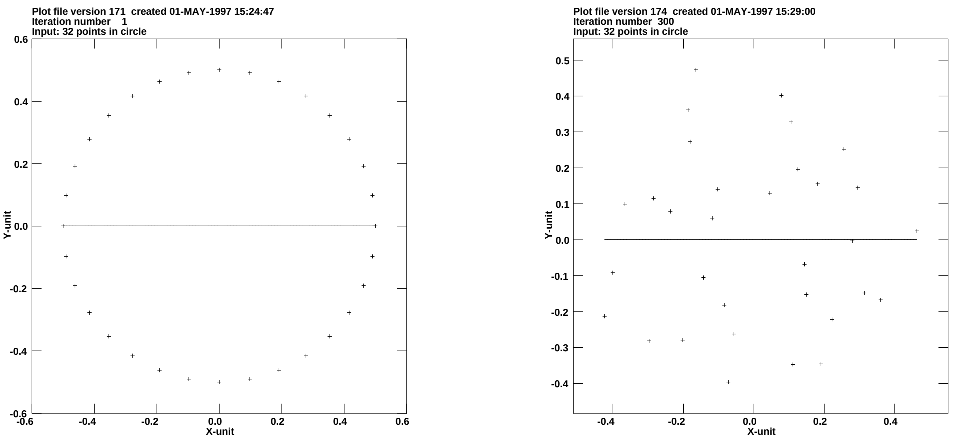

Figure 12: Initial and final configuration of the 32 element array

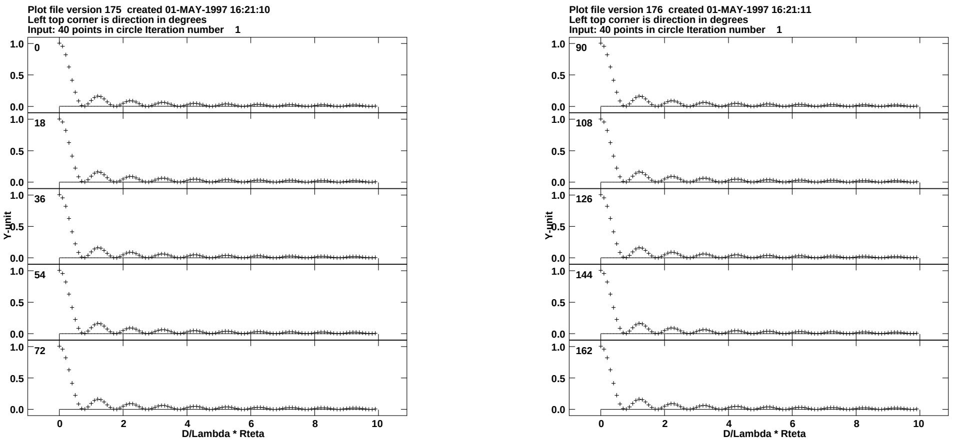

Figure 13: Beams pattern of the initial 40 elements array configuration

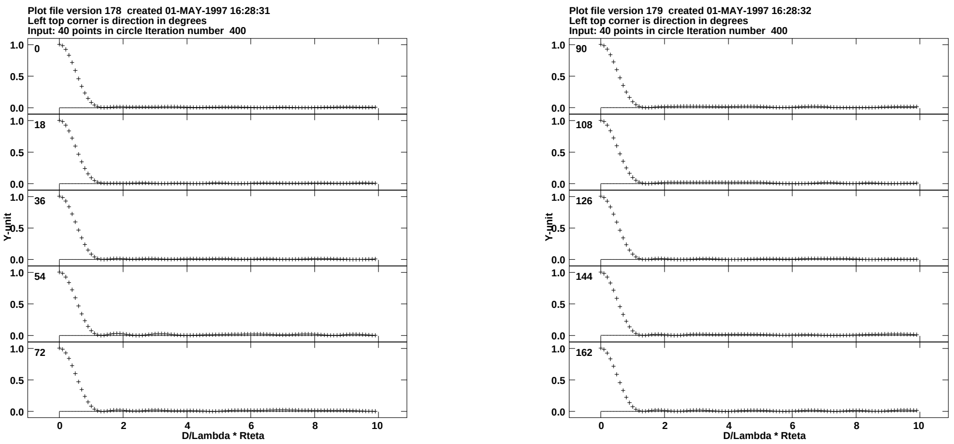

Figure 14: Beams partern of the final 40 elements array configuration

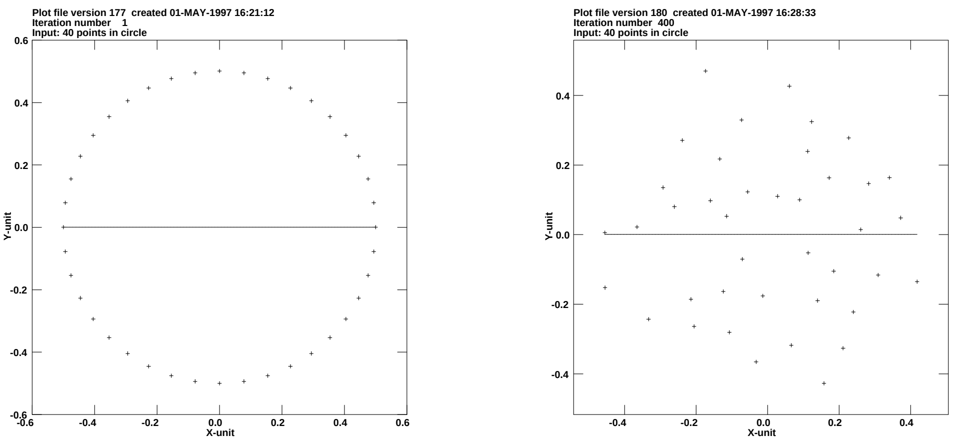

Figure 15: Initial and final configuration of the 40 element array

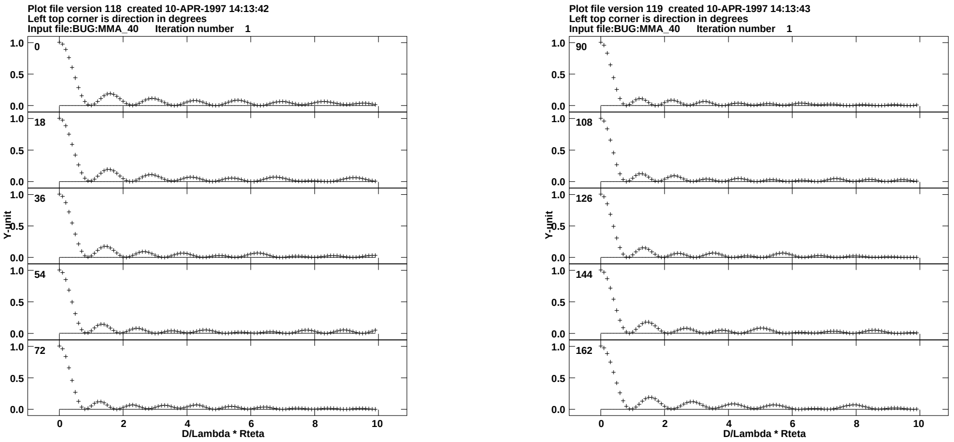

Figure 16: Beams partern of the initial 40 elements array configuration offered by M. Holdaway for Mouna Kea

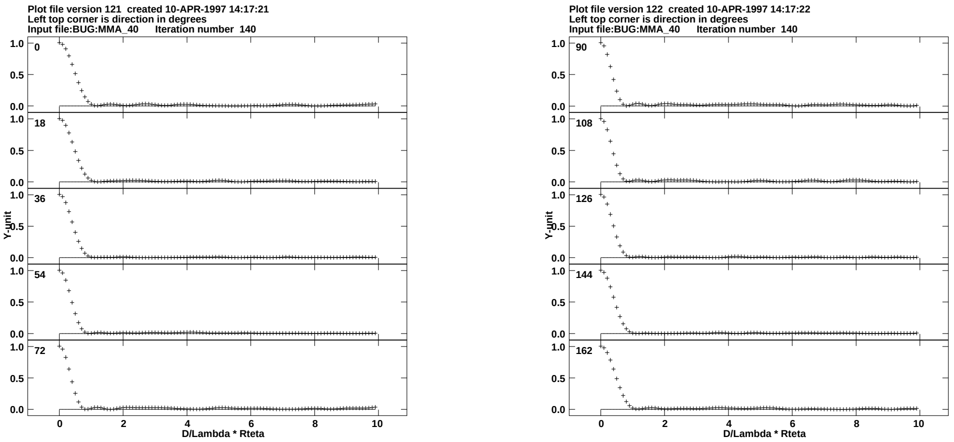

Figure 17: Beams pattern of the final configuration

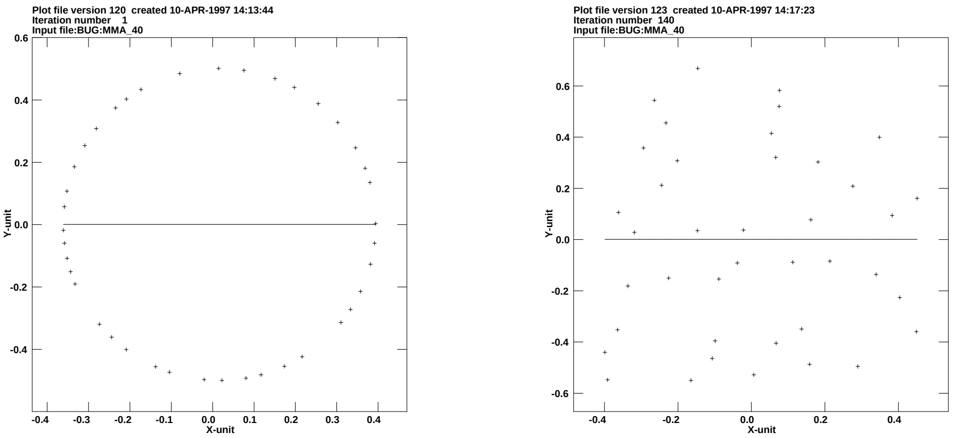

Figure 18: Initial and final configuration of the 40 element array

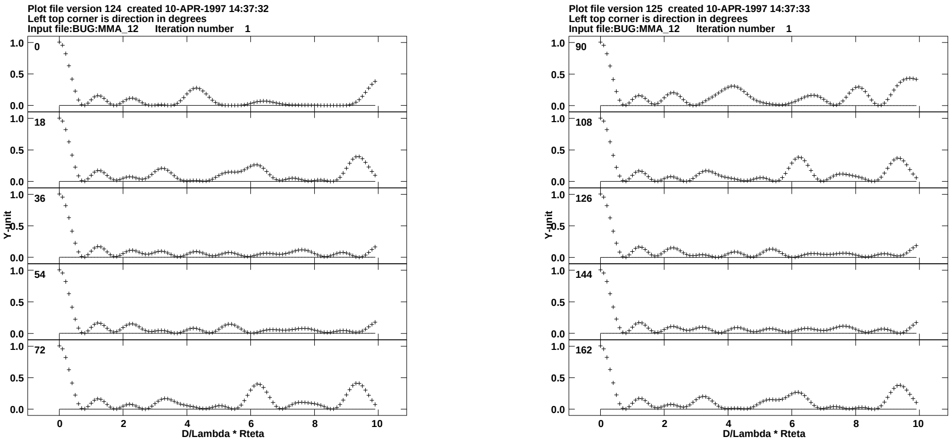

Figure 19: Beams pattern of the initial 12 elements array configuration offered by T. Cornwell as an optimal one by criteria of maximum distance between the U, V points

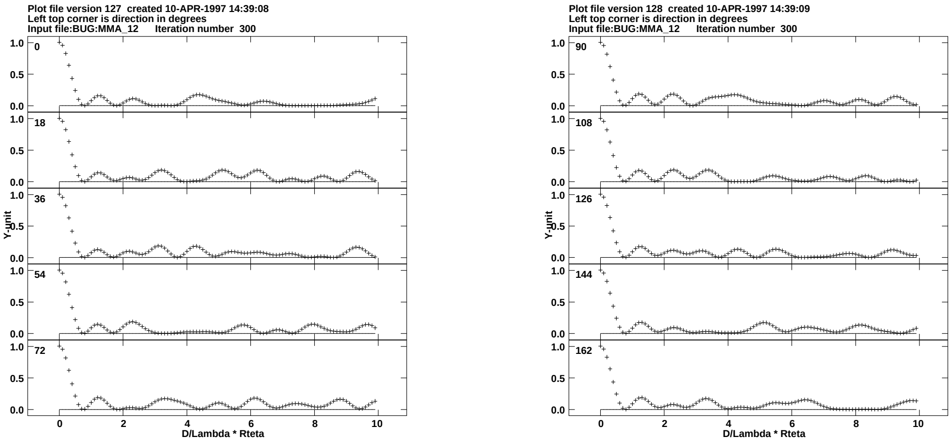

Figure 20: Beams pattern of the final 12 elements array configuration

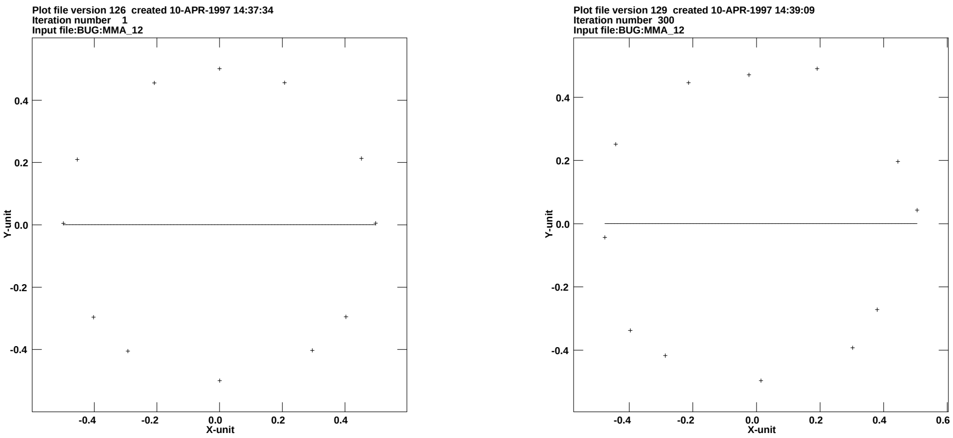

Figure 21: Initial and final configuration of the 12 element array