ALMA Memo 366
A Telescope Pointing Algorithm for ALMA

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Abstract

The stringent pointing specification of $0.''6$ for the ALMA antennas places some strict requirements on the ALMA pointing model formalism. In this document we:

- Describe the basic pointing model formalism used at most radio telescope observatories;
- Present a detailed analysis of the atmospheric refraction correction used in this model;
- Suggest a form for the refraction correction and associated weather parameter measurement sensitivity which will meet the ALMA pointing requirement;
- Suggest a structure for the pointing coefficient information for each antenna.

Contents

1 Primary Pointing Equations

The basic pointing model used at most radio telescopes is a variant of the model described by Stumpff (1972) and Ulich (1981) (for parallel work on pointing models for optical telescopes, see Wallace (1975)). The general philosophy at the heart of these models is that the model should describe real effects, such as axis misalignments and flexures. Empirical functions should only be used to remove any remaining systematic errors, which themselves should have some physical association with the telescope. An alternate school of thought advocates the use of empirical functions, such as spherical harmonics, to describe the pointing model. In the following we describe the physical model for telescope pointing.

The azimuth and elevation terms used to correct the nominal source positions ($A_{command}$ and $E_{command}$, respectively) are given by the equations

$$\begin{aligned} \Delta A &= IA + CA\sec(E) + NPAE\tan(E) + AN\tan(E)\sin(A) \\ &\quad -AW\tan(E)\cos(A) + A_{obs}\sec(E) \qquad (1) \\ \Delta E &= IE + ECEC\cos(E) + AN\cos(A) + AW\sin(A) + E_{obs} + R(P_s, T_s, RH, E) \qquad (2) \end{aligned}$$

where A and E are the azimuth and elevation of the source and the individual pointing coefficients are defined in Table 1. The azimuth and elevation of the target source is then given by

$$\begin{aligned} A_{encoder} &= A_{command} + \Delta A \qquad (3) \\ E_{encoder} &= E_{command} + \Delta E \qquad (4) \end{aligned}$$

The terms IA, CA, NPAE, AN, AW, IE, and ECEC are derived from a collection of 100 or more pointing measurements. These measurements must be representative of the telescope pointing behavior over all azimuth and elevation to allow a linear least-squares fit to Equations 1 and 2. The terms A_{obs} and E_{obs} are observer-applied corrections based on individual pointing measurements made during an astronomical observing program. The refraction correction term in Equation 2 is usually applied on-line for each position command to the telescope. Several approaches to the derivation of this correction have been made and used at existing radio telescopes, some of which we describe in §4.

2 Secondary Pointing Equations

In addition to the primary pointing equations, a set of secondary pointing equations are often necessary to correct for the off-axis placement of receivers. In azimuth, these corrections are given by the equation

$$\Delta A_1 = IA_1 + CA_1\sec(E), \qquad (5)$$

where the left-hand side of the equation corresponds to the "cross-elevation" azimuth correction on the sky, given at a particular elevation by the observer-applied azimuth correction. In elevation, the secondary correction is

Table 1: Basic Telescope Pointing Model Terms

Term[1]	Physical Meaning
ΔA	Total azimuth encoder correction
IA	Azimuth encoder zero offset
CA	Collimation error of the electromagnetic axis
NPAE	Non-perpendicularity between the mount azimuth and elevation axes
AN	Azimuth axis offset/misalignment north-south
AW	Azimuth axis offset/misalignment east-west
A_{obs}	Observer-applied azimuth correction
ΔE	Total elevation encoder correction
IE	Elevation encoder zero offset
ECEC	Gravitational flexure correction at the horizon
E_{obs}	Observer-applied elevation correction
R(P_s,T_s,RH,E)	Atmospheric refraction correction, which is a function of ambient pressure (P_s), temperature (T_s), relative humidity (RH), and the elevation of the source

[1] We use the notation developed in the TPOINT pointing analysis program.

$$\Delta E_1 = IE_1 + ECEC_1 \cos(E), \tag{6}$$

These terms correspond to the linear (in $\cos(E)$) terms in the principal pointing equations given above: IA_1 is the supplemental azimuth encoder offset, CA_1 is the supplemental electromagnetic collimation error correction, IE_1 is the supplemental elevation encoder offset, and $ECEC_1$ is the supplemental gravitational bending error.

At the 12 Meter Telescope, each of the four receiver bays have their own set of these secondary coefficients to compensate for slight differences between feed and mirror alignments in each bay. The intent of these pointing coefficients is to keep the required observer-applied pointing offsets as close to zero as possible. The determination of these coefficients requires far fewer measurements than the main pointing equations and can thus be adjusted on a more frequent basis.

3 Additional Pointing Equation Terms

As the need arises, additional pointing equation terms can be added to describe additional physical deformations of a telescope structure. Some of the additional terms used in radio telescope pointing models are:

- *Track Model:* For radio telescopes with a wheel-and-track azimuth mount, it is often necessary to correct for irregularities in the azimuth track as a function of azimuth. The JCMT includes such a correction in their pointing model.

- *Additional Gravitational Flexure:* The complete expression for the gravitational deformation (Hooke's law) of an antenna includes a term in $\sin(E)$. The neglect of this term is correct for centro-symmetric telescope designs (which includes the vast majority of radio telescopes). If the elevation drive/encoder system of the telescope is miscentered, or if it is determined that non-Hooke's-law deflections are present, it might be necessary to include this or other additional terms to fully describe the gravitational flexure.

- *Bearing Warp:* If an azimuth or elevation bearing is warped, it may be necessary to add terms in $\sin(nA)$ and $\cos(nA)$ to a pointing model, where n is an integer ≥ 2. Such a correction is necessary for a more complete analysis of the 12 Meter Telescope pointing behavior.

4 The Refraction Correction

The astronomical refraction R is defined as the difference between the true and apparent elevation of an astronomical object, and is given by the following equation:

$$R = \int_1^{n_0} \frac{\cot(E)}{n} dn. \tag{7}$$

where n is the index of refraction and the integral is carried out along the path of the signal. From Snell's law,

$$nr\cos(E) = n_0 r_0 \cos(E_0), \tag{8}$$

in which r is the geocentric distance to a point in the atmosphere and n_0, r_0, and E_0 are the measured refractive index, geocentric distance, and source elevation at the location of the observatory. Combining Equations 7 and 8 we obtain the general expression for the astronomical refraction in a spherically-symmetric atmosphere:

$$R = \int_1^{n_0} \frac{n_0 r_0 \cos(E_0)}{n(n^2r^2 - n_0^2 r_0^2 \cos^2(E_0))^{\frac{1}{2}}} dn. \tag{9}$$

At this point, a series expansion is usually made to derive an analytical form for R which depends on elevation and the local physical conditions at the observatory. In general, R is split into two terms: a meteorological-dependent term which is a function of only the local atmospheric physical conditions and a term which is primarily elevation-dependent, but also contains some dependence on the local atmospheric physical conditions:

$$R(P_s, T_s, RH, E) = R_0(P_s, T_s, RH) f(P_s, T_s, RH, E). \tag{10}$$

where P_s, T_s, and RH are the local atmospheric pressure, temperature, and relative humidity, and E is the source elevation. In order to derive R(P_s,T_s,RH,E) we need to derive the meteorological dependence of the refractivity N_0, which is defined as

$$\begin{aligned} N_0 &= (n-1) \times 10^6 \\ &= R_0(radians) \times 10^6 \\ &= \frac{R_0(arcsec)}{0.206265} \end{aligned} \tag{11}$$

where n is the complex index of refraction and R_0 is the meteorological-dependent refraction coefficient, which is nearly independent of wavelength. Based on atmospheric studies, several forms for N_0 valid at radio wavelengths, N_0^{rad}, have been derived. Two forms for N_0^{rad} which are good to 0.5% at frequencies below 100 GHz are given by Brussaard & Watson (1995) (see also Crane (1976) and Liebe & Hopponen (1977))

$${}^{BW}N_0^{rad} = 77.6\frac{P_s}{T_s} - 5.6\frac{P_w}{T_s} + 3.75 \times 10^5 \frac{P_w}{T_s^2} \quad ppm \tag{12}$$

and Smith & Weintraub (1953)

$${}^{SW}N_0^{rad} = 77.6\frac{P_s}{T_s} - 12.8\frac{P_w}{T_s} + 3.776 \times 10^5 \frac{P_w}{T_s^2} \quad ppm \tag{13}$$

where

P_d is the partial pressure of dry gases in the atmosphere (in mb),

P_w is the partial pressure of water vapor at the surface (in mb),

P_s is the total surface barometric pressure (in mb), which is equal to $P_d + P_w$, and

T_s is the surface ambient air temperature (in Kelvin).

P_s must be measured by a barometer at the telescope site. P_w may be calculated from the expression

$$P_w = RH\frac{e_{sat}}{100}, \tag{14}$$

where RH is the surface relative humidity (in percent) and e_{sat} is the surface saturated water vapor pressure (in mb) and is given by Crane (1976) as

$$e_{sat} = 6.105 \exp\left[25.22\left(\frac{T_s - 273}{T_s}\right) - 5.31 \log_e\left(\frac{T_s}{273}\right)\right] \tag{15}$$

and is given by Buck (1981) as

$$e'_w = e_{sat} = \left(1.0007 + 3.46 \times 10^{-6} P_s\right) 6.1121 \exp\left[\frac{17.502(T_s - 273.15)}{T_s - 32.18}\right]. \tag{16}$$

The analysis presented below was done using the Crane (1976) relation for e_{sat}, as the Buck (1981) relation was only made aware to us later. Comparison of the Crane (1976) and Buck (1981) relations for e_{sat} over the ranges of pressure and temperature typical of

the Chajnantor site indicate a less than 1.2% difference. Therefore, use of either relation for e_{sat} appears to be satisfactory.
Note that in our analysis of N_0^{rad} we have ignored the partial pressure contribution due to carbon dioxide (only $\sim$ 0.03% of the total pressure) and that the pressures and temperatures are as defined above. A similar expression for N_0^{rad} to the three given above can also be found in Allen (1973).

In the following, we examine some of the commonly-used approximations to Equation 9 and suggest a form which might be appropriate for the ALMA antennas on the Chajnantor site.

4.1 Ulich Model

Ulich (1981) describes a form for the astronomical refraction appropriate to observations at millimeter wavelengths which is separable into two terms:

$$R(E) = R_0^{rad}(P_s, T_s, RH) f(E). \tag{17}$$

Taking the Brussaard & Watson (1995) relation (Equation 12) for N_0^{rad} and solving for R_0^{rad} in arcseconds yields

$$R_0^{rad} = 16.01\frac{P_s}{T_s} - 1.15\frac{P_w}{T_s} + 7.734937 \times 10^4 \frac{P_w}{T_s^2}, \tag{18}$$

Note that Ulich (1981) quotes a similar expression for $\log(e_{sat})$ as that given in Equation 15, but does not give a reference.

The elevation dependence of the refraction correction is given by

$$f(E) = \frac{\cos(E)}{\sin(E) + 0.00175\tan(87.5 - E)}. \tag{19}$$

Ulich (1981) states that this form for the elevation dependence of the refraction has an uncertainty of $2''$ for $E \geq 3°$.

4.2 IRAM/JCMT Model

At the JCMT and the IRAM telescopes, an adaptation of the formalism given in Smart (1977) is used. Assuming that the height of the atmosphere (r) is small in comparison to the radius of the Earth (r_0), or:

$$\frac{r}{r_0} = 1 + \epsilon \tag{20}$$

where $\epsilon \ll 1$, then Equation 9 becomes:

$$R = \int_1^{n_0} \frac{n_0 \cos(E_0)}{n(n^2 - n_0^2\cos^2 E)^{\frac{1}{2}}} dn - \int_1^{n_0} \frac{n_0 \cos(E_0) n\epsilon}{n(n^2 - n_0^2 \cos^2(E))^{\frac{3}{2}}} dn \tag{21}$$

which, becomes:

$$R = A\cot(E) + B\cot^3(E) + C\cot^5(E) \tag{22}$$

where A, B, and C are constants dependent on the local atmospheric temperature, pressure, and relative humidity. The approximations used to derive Equation 22 are good for $E \geq 15°$.

At the JCMT, the A and B terms (the C term is taken to be 0) have been derived by taking the basic variations of temperature, density, and pressure with altitude predicted by the American Standard Atmosphere (ASA) as given by Allen (1973). Slight local variations from the ASA predictions based on studies of the Hawaiian atmosphere by Takahashi (University of Hawaii, Hilo) further constrain these derivations. See Iain Coulson's 1987 and 1988 documents (available from the JCMT web page) for further information on the exact form for these coefficients.

4.3 The Proposed ALMA Model

The ALMA antenna pointing specification requires a relative pointing accuracy of less than $0.6''$. Therefore, a more accurate model for the atmospheric refraction must be developed. This means that a more accurate representation of the elevation-dependence and a better description of the meteorological-dependence of R(E) must be derived. Yan (1996) has used the generator function method of atmospheric refractive integrals to formulate R(E). Assuming an exponential atmospheric profile, the atmospheric refractivity is given by

$$N(h) = N_0 \exp\left(\frac{-(h-h_0)}{H}\right) \tag{23}$$

where h and h_0 are height coordinates and H is the effective height of the atmosphere

$$H = \frac{RT}{M_0 g} \tag{24}$$

where R is the universal gas constant, M_0 is the molar mass of the atmosphere, T is the temperature of the atmosphere, and g is the gravitational acceleration constant measured at the center of the vertical column of air. The astronomical refraction can be approximated using the generator function formalism (see Yan & Ping (1995) and Yan (1996) for details) as follows:

$$R(E) = R_0 \cos(E) m'(E) \tag{25}$$

where

$$m'(E) = \frac{1}{\sin(E) + \frac{A_1}{I^2\csc(E) + \frac{A_2}{\sin(E) + \frac{13.24969}{I^2\csc(E)+173.4233}}}} \tag{26}$$

and

$$I = \sqrt{\frac{r_0}{2H}}\tan(E) \tag{27}$$

For the meteorological-dependent terms A_1 and A_2, Yan (1996) found that, for a wide range of meteorological conditions at radio frequencies:

$$\begin{aligned} A_1^{rad} &= 0.5753868 + 0.5291 \times 10^{-4}(P_s - 1013.25) - 0.2819 \times 10^{-4} P_w - \\ &\quad 0.9381 \times 10^{-6} P_w^2 - 0.5958 \times 10^{-3}(T_s - 258.15) + \\ &\quad 0.2657 \times 10^{-5}(T_s - 258.15)^2 \\ A_2^{rad} &= 1.301211 + 0.2003 \times 10^{-4}(P_s - 1013.25) - 0.7285 \times 10^{-4} P_w + \\ &\quad 0.2579 \times 10^{-5} P_w^2 - 0.2595 \times 10^{-2}(T_s - 258.15) + \\ &\quad 0.8509 \times 10^{-5}(T_s - 258.15)^2 \end{aligned} \tag{28}$$

and at optical frequencies (since we will likely use optical pointing as an aid in deriving the telescope pointing model)

$$\begin{aligned} A_1^{opt} &= 0.5787089 + 0.5609 \times 10^{-4}(P_s - 1013.25) - 0.6229 \times 10^{-3}(T_s - 258.15) + \\ &\quad 0.2824 \times 10^{-5}(T_s - 258.15)^2 + 0.5177 \times 10^{-3} P_w + \\ &\quad 0.29 \times 10^{-6} P_w^2 - 0.1644 \times 10^{-1}(\lambda - 0.532) + 0.491 \times 10^{-1}(\lambda - 0.532)^2 \\ A_2^{opt} &= 1.302474 + 0.2142 \times 10^{-4}(P_s - 1013.25) + 0.1287 \times 10^{-2} P_w + \\ &\quad 0.65 \times 10^{-6} P_w^2 - 0.6298 \times 10^{-2}(\lambda - 0.532) + 0.189 \times 10^{-1}(\lambda - 0.532)^2 - \\ &\quad 0.2623 \times 10^{-2}(T_s - 258.15) + 0.8776 \times 10^{-5}(T_s - 258.15)^2 \end{aligned} \tag{29}$$

where T_s, P_s, and P_w are defined in §4 (note that we have converted the temperatures in this equation from Celsius to Kelvin ($T_C = T_K + 273.15$)). Equation 25 with A_1 and A_2 given by Equations 28 is purported by Yan (1996) to be accurate to better than 0.″3 for E>2°. For E>15°, the radio refraction can be calculated to an accuracy of better than 0.″2 by using the following constants:

$$\begin{aligned} A_1^{rad} &= 0.6306849 + 0.6069 \times 10^{-4}(P_s - 1013.25) - 0.2532 \times 10^{-4} P_w - \\ &\quad 0.9881 \times 10^{-6} P_w^2 - 0.5154 \times 10^{-3}(T_s - 258.15) + \\ &\quad 0.2880 \times 10^{-5}(T_s - 258.15)^2 \\ A_2^{rad} &= 1.302642 \end{aligned} \tag{30}$$

and the optical refraction can be calculated to this accuracy with the following constants

$$\begin{aligned} A_1^{opt} &= 0.6345042 + 0.6430 \times 10^{-4}(P_s - 1013.25) - 0.5420 \times 10^{-3}(T_s - 258.15) + \\ &\quad 0.3011 \times 10^{-5}(T_s - 258.15)^2 + 0.5314 \times 10^{-2} P_w + \\ &\quad 0.97 \times 10^{-6} P_w^2 - 0.1891 \times 10^{-1}(\lambda - 0.532) + \\ &\quad 0.565 \times 10^{-1}(\lambda - 0.532)^2 \\ A_2^{opt} &= 1.302642 \end{aligned} \tag{31}$$

A comparison of the coefficients listed in Equations 28 through 31 with various atmospheric models indicates that they are valid over a wide range of physical conditions (Yan (1996)).

4.4 The Refractivity at the Observatory R_0

4.4.1 Radio

The refractivity at the observatory, R_0^{rad}, is given by Equation 11. Since Equations 12 and 13 are quoted to be good only up to 100 GHz, we have investigated their validity up to 1000 GHz. Using the atmospheric model described in Liebe *et al.* (1993) with the measured ranges for atmospheric pressure (548 to 560 mb), temperature (-20 to 20 C), and relative humidity (0 to 100%) for the ALMA site near Cerro Chajnantor, we have compared N_0^{rad} calculated from the Liebe *et al.* (1993) model with those calculated using Equations 12 and 13. Differences between the values for N_0^{rad} derived from 12 and 13 differ by less than 0.1%. Comparisons of N_0^{rad} calculated using Equation 12 with N_0^{rad} derived from the Liebe *et al.* (1993) model for two extreme sets of physical conditions are given in Figures 1 and 2. Figures 1 and 2 show the percentage difference between the two estimates for N_0^{rad} under "worst case" (maximum humidity, maximum temperature, minimum pressure) and "best case" (minimum humidity, minimum temperature, maximum pressure) conditions. The largest departures of ${}^{BW}N_0^{rad}$ from the Liebe *et al.* (1993) model predictions occur near telluric O_2 and H_2O transitions. Figures 1 and 2 also show the frequency ranges for the 10 ALMA frequency bands (Emerson (1999)). The percentage difference between ${}^{SW}N_0^{rad}$ and N_0^{rad} derived from the Liebe *et al.* (1993) model at the edges of these receiver bands are listed in Table 2. The maximum difference occurs at 500 GHz, where the 11% error in the ${}^{BW}N_0^{rad}$ calculation will translate into a 2% error in R_0^{rad} (see Equation 11). Since typical values for R(E) are around 30″, a maximum 2% error in R_0^{rad} is significant given the ALMA pointing specification of 0.″6, it is probably prudent to use the full Liebe *et al.* (1993) model to calculate N_0^{rad}. For less stringent pointing specifications, any of the three relations for N_0^{rad} listed in Equations 12 and 13 will suffice.

4.4.2 Optical

Refractivity in the optical is cast in a slightly different form than that in the radio due to the fact that at optical wavelengths refractivity is no longer frequency independent. Birch & Downs (1993) (see also Livengood *et al.* (1999)) state that the optical refractivity is given by the following:

$$N_0^{opt} = N_{STP} \times N_{TP} - N_{RH} \tag{32}$$

where

$$N_{STP} = 83.4305 + \frac{24062.94}{130 - \lambda^{-2}} + \frac{159.99}{38.9 - \lambda^{-2}} \tag{33}$$

$$N_{TP} = \frac{P_s}{1.01325 \times 10^3} \frac{(273.15 + 15)}{T_s} \frac{\left[1 + (3.25602 - 0.00972 T_s) P_s \times 10^{-3}\right]}{1.00047} \tag{34}$$

$$N_{RH} = P_w \times (37.345 - 0.401\lambda^{-2}) \times 10^{-3} \tag{35}$$

with P_s and P_w in mb, T_s in K, and λ in micron. Note that we have ignored the small correction for an increase in CO_2 concentration in Equation 32.

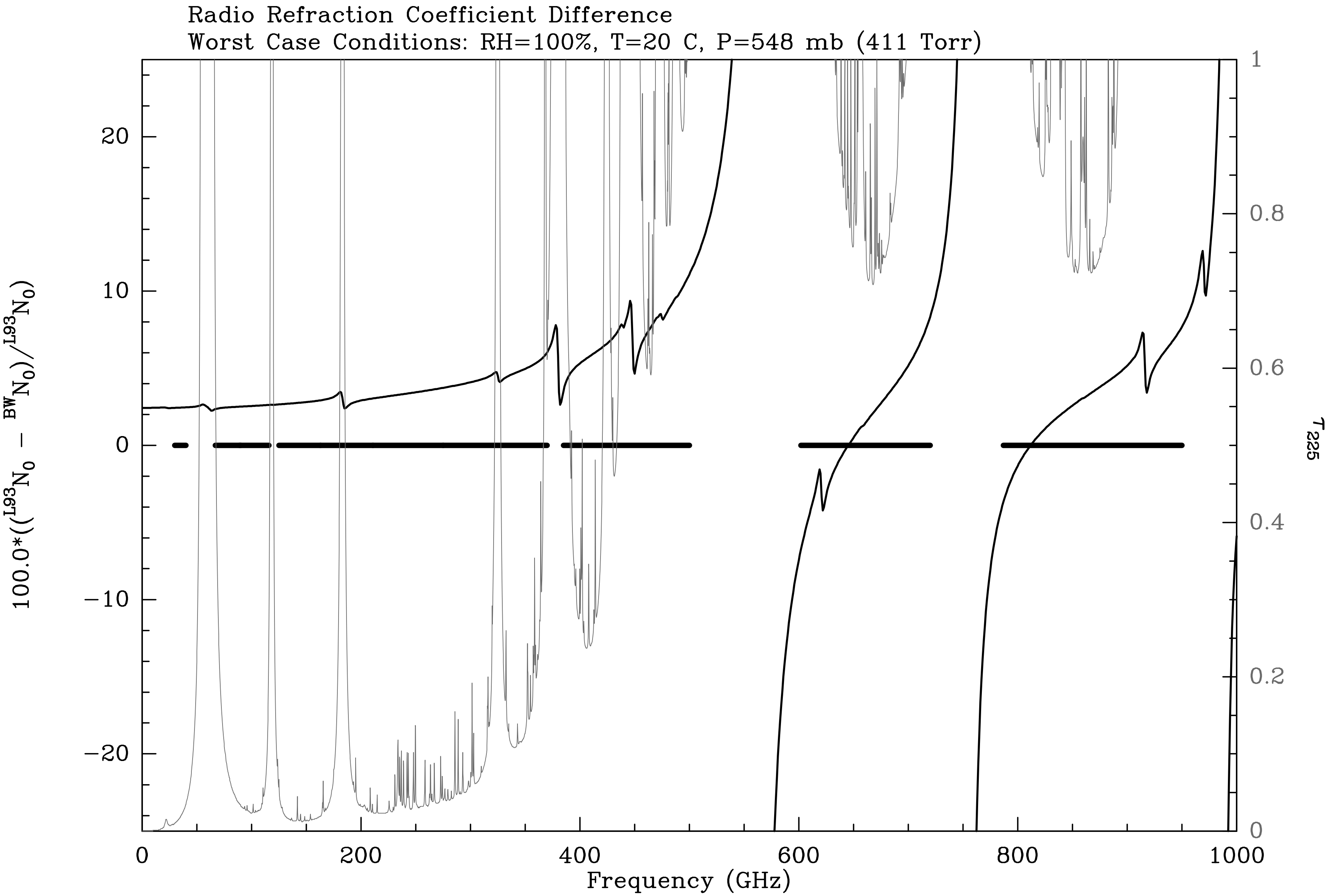

Figure 1: Radio refraction coefficient difference (dark solid curve) between the Brussaard & Watson (1995) and Liebe *et al*. (1993) estimates for N_0^{rad} under the most extreme set of physical conditions: (P_s,T_s,RH) = (548 mb, 20 C, 100%) measured at the ALMA (Cerro Chajnantor) site. The light dotted curve shows the atmospheric opacity assuming 1mm PWV at the ALMA site. The solid horizontal lines show the frequency ranges for the 10 ALMA receiver bands (Emerson (1999)).

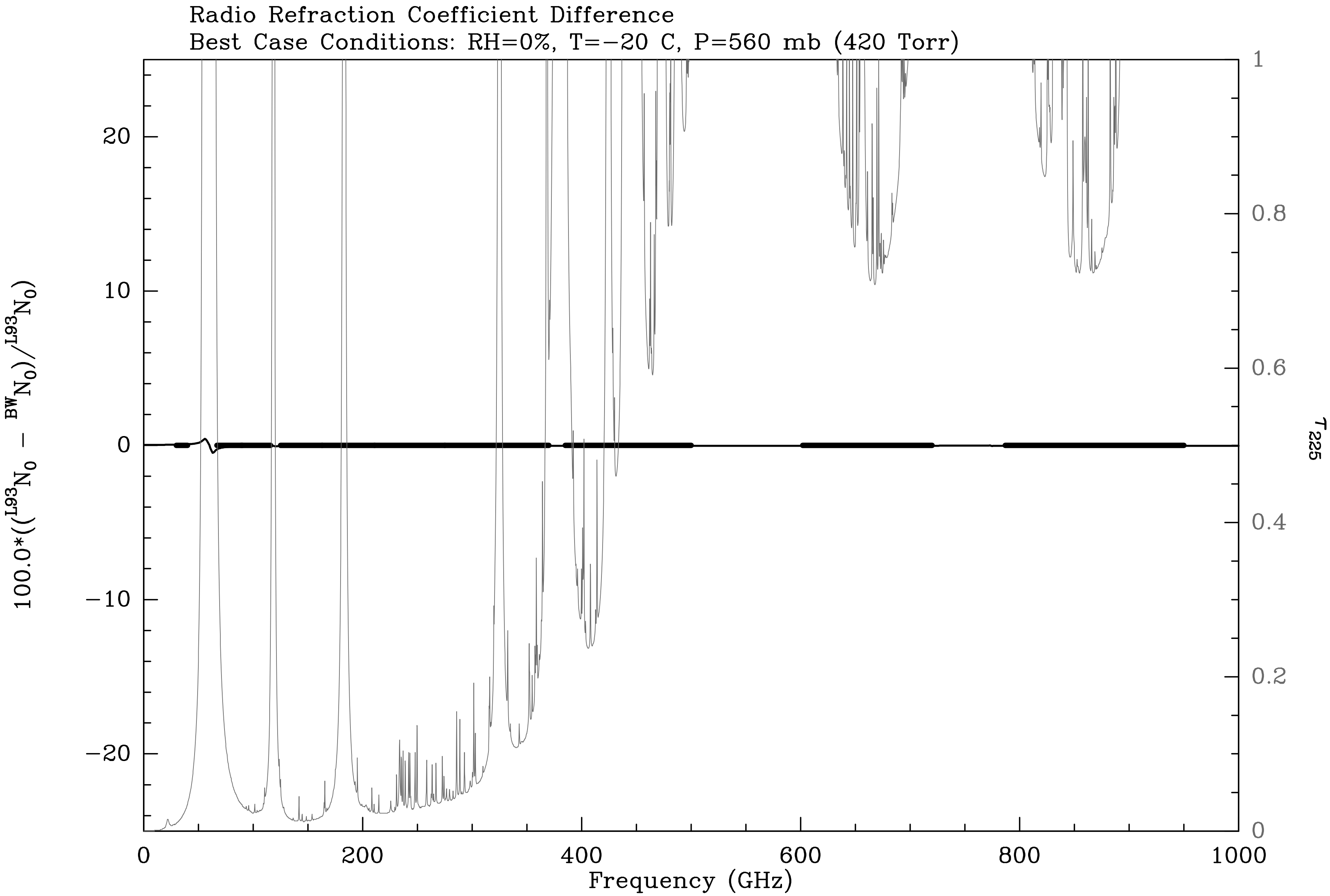

Figure 2: Radio refraction coefficient difference (dark solid curve) between the Brussaard & Watson (1995) and Liebe *et al.* (1993) estimates for N_0^{rad} under the most benign set of physical conditions: (P_s,T_s,RH) = (560 mb, -20 C, 0%) measured at the ALMA (Cerro Chajnantor) site. The light dotted curve shows the atmospheric opacity assuming 1mm PWV at the ALMA site. The solid horizontal lines show the frequency ranges for the 10 ALMA receiver bands (Emerson (1999)).

Table 2: N_0^{rad} for the ALMA Receiver Band Edges[1]

Receiver	Band Edge (GHz)	${}^{L93}N_0^{rad}$ (ppm)	$100 \times \frac{({}^{L93}N_0^{rad} - {}^{BW}N_0^{rad})}{{}^{L93}N_0^{rad}}$
1	30	253.07	2.43
1	40	253.15	2.46
2	67	252.86	2.34
2	90	253.28	2.50
3	89	253.27	2.50
3	116	253.57	2.61
4	125	253.65	2.65
4	163	254.34	2.91
5	211	254.66	3.03
6	275	256.47	3.72
7	370	262.72	6.01
8	385	255.93	3.51
8	500	277.67	11.07
9	602	231.29	-6.76
9	720	269.34	8.32
10	787	238.18	-3.67
10	950	267.42	7.66

[1] Assumed "worst case" physical conditions: (P_s,T_s,RH) = (548 mb, 20 C, 100%). ${}^{BW}N_0^{rad}$ = 246.93 ppm for these physical conditions.

4.4.3 Refractivity Comparison

A comparison between R calculated using the proposed ALMA model (both radio and optical) and that presented in Ulich (1981) is shown in Figure 3. The Brussaard & Watson (1995) relation for N_0^{rad} was used in both R calculations, while N_0^{opt} has been calculated using Equation 32 assuming $\lambda = 0.6\mu m$. Note that this comparison does not accurately represent the true uncertainty in each calculation, which is $2''$ for the Ulich (1981) model and $0.''3$ for the proposed ALMA model.

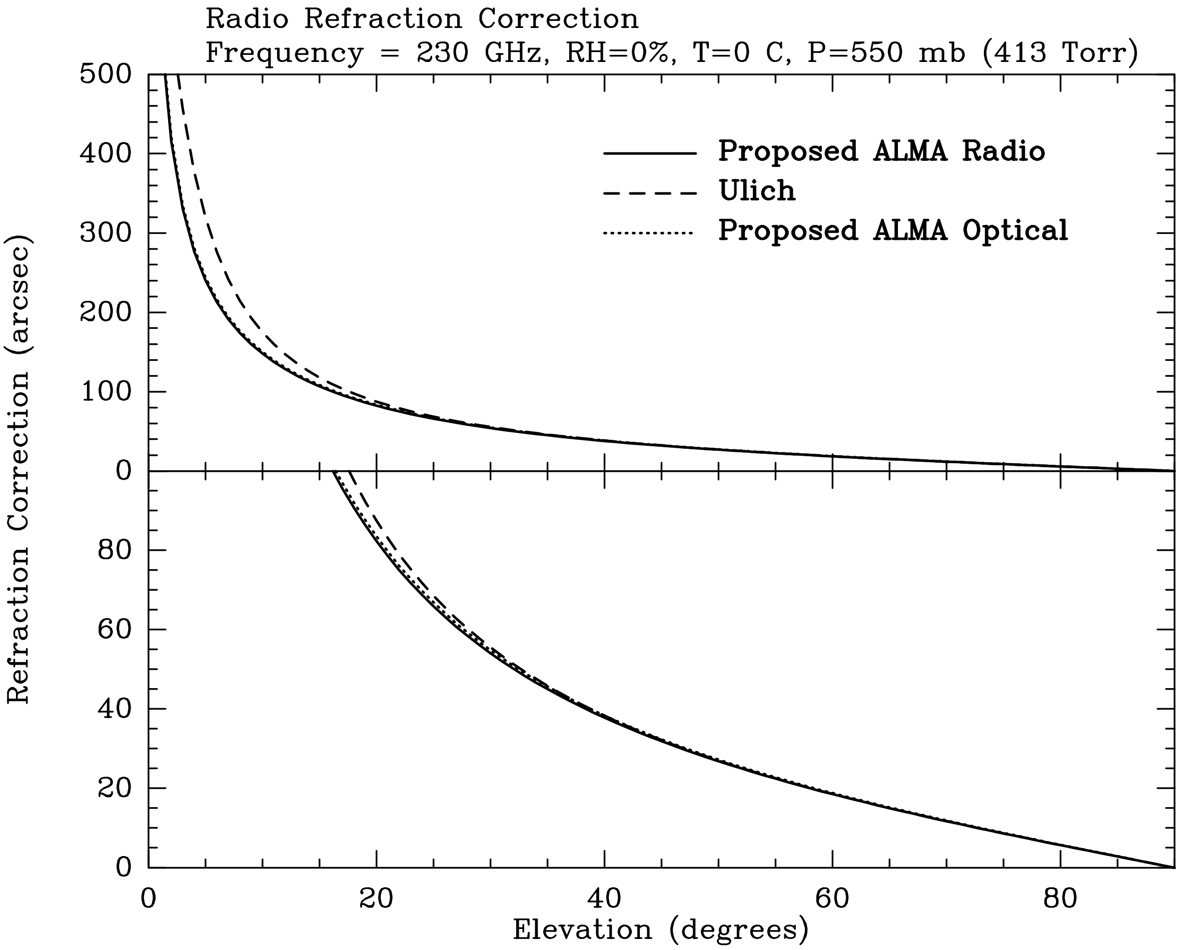

Figure 3: Total radio refraction correction for the conditions indicated assuming the proposed ALMA model (radio: solid curve, optical: dotted curve) and the Ulich (1981) model (dashed curve).

4.5 Weather Parameter Measurement Requirements

In the analysis above we have assumed perfect measurements from our weather instruments. In fact, weather instrumentation has inherent uncertainties. Table 3 lists the necessary measurement accuracies for each kind of weather instrument. To arrive at these requirements, we analyzed the affects which variations in RH, T_s, and P_s have on calculations

Table 3: Surface Weather Measurement Requirements for Refraction Calculations

Parameter	Symbol	Required Accuracy	Note
Barometric Pressure	P_s	0.5 mb	0.017 Hz (1 minute) sampling
Ambient Temperature	T_s	0.1 C	0.017 Hz (1 minute) sampling
Relative Humidity	RH	0.5%	0.017 Hz (1 minute) sampling
Wind Speed	W_s	0.5 m/s	1-10 Hz sampling
Wind Direction	W_d	5 deg	1-10 Hz sampling

[1] Instrument tower height assumed to be 10m.

of the radio refraction correction R. Figures 4 and 5 present the results from this analysis for average ((P_s,T_s,RH)=(550 mb, 0 C, 0%)) and the most extreme ((P_s,T_s,RH)=(548 mb, 20 C, 100%)) sets of meteorological conditions. In Figure 4, for example, overestimating the RH by 0.5% leads to an error of $\approx 0.''25$ at 10°elevation.

5 Pointing Data Analysis

As described in §1, the primary pointing terms IA, CA, NPAE, AN, AW, IE, and ECEC are derived from a linear least-squares fit to a collection of 100 or more pointing measurements. It is important for these pointing measurements to be representative of a given telescope's pointing behavior over all azimuth and elevation. A typical procedure for analyzing a set of pointing measurements is the following:

1. If doing interferometric pointing (at least 3 antennas required):
   (a) Solve for the N antenna-based amplitudes from individual baseline-based total power measurements.
   (b) In the least-squares fitting analysis described below, N independent matrix inversions must be made in order to derive the pointing coefficients for N antennas.
2. Cull erroneous pointing fit results from your pointing data set. This is best done using a database system which can be used to select data based on fit quality.
3. Export the resultant fit results to a file which can be analyzed by a linear least-squares fitting program. It is also advantageous that the analysis program have informative plotting capabilities and the ability to add or subtract terms to the pointing equations being fit.
4. Once a satisfactory fit to the pointing measurements is obtained, apply the new coefficients to the telescope control program and check to see that the antennas now acquires positions correctly.

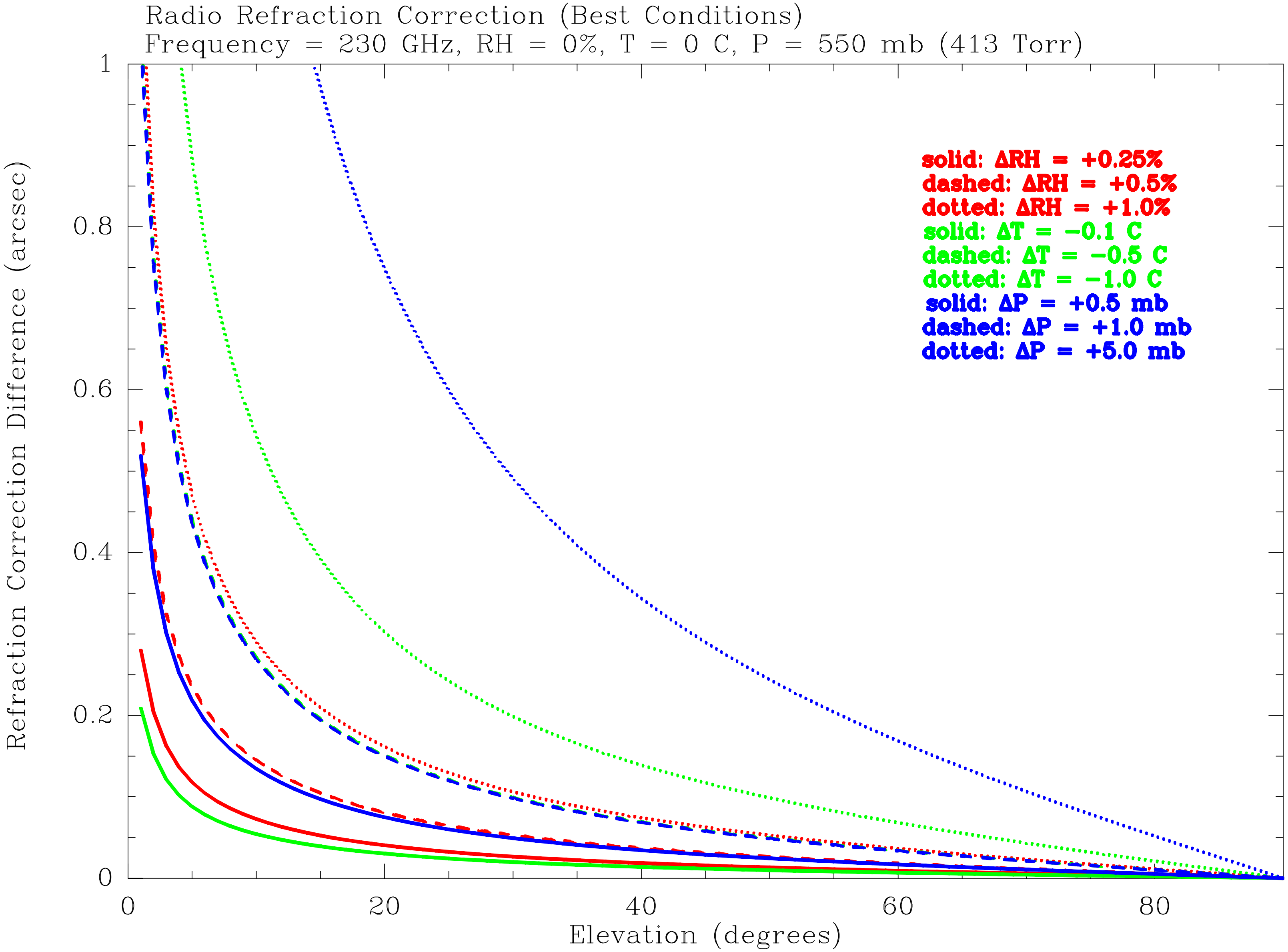

Figure 4: Radio refraction correction difference under typical meteorological conditions on the Chajnantor site for representative errors in RH (red), T_s (green), and P_s (blue). For example, an error in measurement of the surface temperature T_s of -1.0 C leads to an error of $+0.3''$ in R at an elevation angle of 20°. Note that the curves for an error in T_s of -0.5 C (dashed green) and an error in P_s of $+1.0$ mb (dashed blue) overlap.

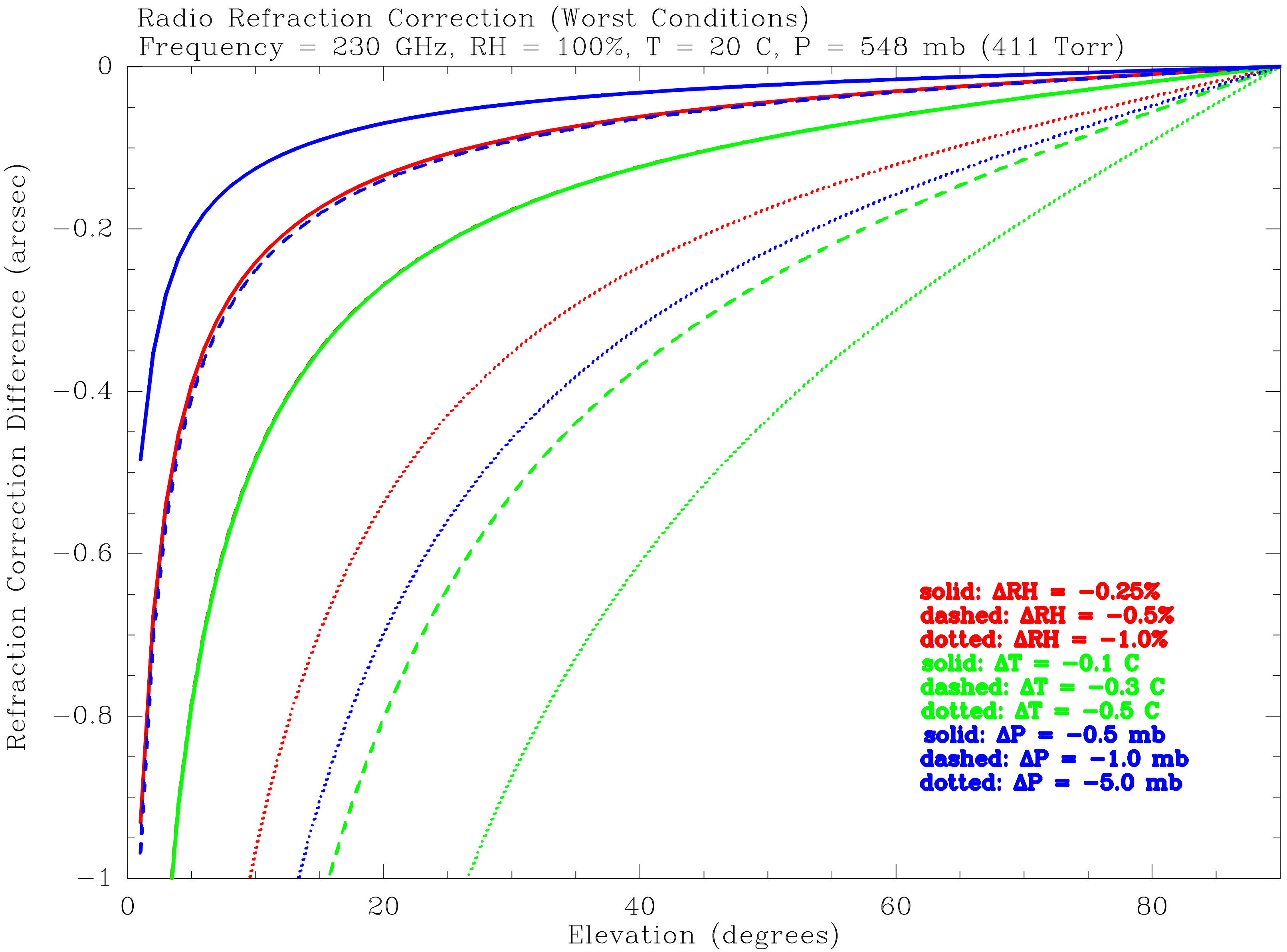

Figure 5: Radio refraction correction difference under the most extreme (worst) meteorological conditions on the Chajnantor site for representative errors in RH (red), T_s (green), and P_s (blue). For example, an error in measurement of the surface temperature T_s of -0.3 C leads to an error of $-0.85''$ in R at an elevation angle of $20°$. Note that the curves for an error in RH of -0.25% (solid red) and an error in P_s of -1.0 mb (dashed blue) overlap somewhat.

For the culling of the pointing data, any SQL-based database program will do. For the linear least-squares fit analysis, we have investigated two commercial possibilities: Mathematica (using procedures developed by Fred Schwab) and TPOINT (by P. T. Wallace). They are both excellent choices, with different strengths and weaknesses. Mathematica allows for a great deal of flexibility as the code is in the form of home-grown scripts, but one must develop these scripts, which must include graphical and user interface capabilities. TPOINT, on the other hand, is a commercial package written and maintained by P. T. Wallace. TPOINT was written specifically for analysis of pointing data. It contains all of the capabilities that one needs to analyze the pointing characteristics of a telescope and does not at this point seem to require modification.

6 Pointing Coefficient File Structure

In the following we list a suggested pointing coefficient file structure. This file is structured such that:

- For maximum flexibility, the pointing coefficient file consists of a list of character strings and numbers. The character strings are the coefficient names (such as "NPAE") and the numbers the coefficients in arcseconds. Comments are designated by a "#" in column 1.
- The model is assumed to be set up dynamically when the system is booted and to contain the full repertoire of terms as defined in the TPOINT program.
- There is no restriction that each antenna has a model of the same form. For the same antenna design most of the terms will be in common, but there's nothing to stop individual antennas having their own peculiarities.
- The "DATE-OBS" line is a machine readable string containing the date/time when the *observations* were made (not when the coefficients were calculated).
- The first 7 coefficients describe the pointing model for a detector located at the boresight of the telescope, even though it is not a requirement for there to actually be a detector at the boresight position.
- Band 0 is the optical pointing telescope.

```
# Pointing Model Coefficients for Antenna n
DATE-OBS 2002-02-20T22:18:39.613

IA              xxx.xx
CA              xxx.xx
NPAE            xxx.xx
AN              xxx.xx
AW              xxx.xx
IE              xxx.xx
```

```
ECEC           xxx.xx
IA0            xxx.xx
CA0            xxx.xx
IE0            xxx.xx
ECEC0          xxx.xx
IA1            xxx.xx
CA1            xxx.xx
IE1            xxx.xx
ECEC1          xxx.xx
IA2            xxx.xx
CA2            xxx.xx
IE2            xxx.xx
ECEC2          xxx.xx
IA3            xxx.xx
CA3            xxx.xx
IE3            xxx.xx
ECEC3          xxx.xx
IA4            xxx.xx
CA4            xxx.xx
IE4            xxx.xx
ECEC4          xxx.xx
IA5            xxx.xx
CA5            xxx.xx
IE5            xxx.xx
ECEC5          xxx.xx
IA6            xxx.xx
CA6            xxx.xx
IE6            xxx.xx
ECEC6          xxx.xx
IA7            xxx.xx
CA7            xxx.xx
IE7            xxx.xx
ECEC7          xxx.xx
IA8            xxx.xx
CA8            xxx.xx
IE8            xxx.xx
ECEC8          xxx.xx
IA9            xxx.xx
CA9            xxx.xx
IE9            xxx.xx
ECEC9          xxx.xx
IA10           xxx.xx
CA10           xxx.xx
IE10           xxx.xx
ECEC10         xxx.xx
```

7 Conclusions

- The pointing model formalism described in §1, coupled with the refraction calculation described in §4.3 which relies upon the weather instrumentation described in §4.5 will allow a characterization of the ALMA antenna pointing behavior with a basic uncertainty of at most $0.''3$ under most meteorological conditions.
- A full atmospheric model, such as that given by Liebe *et al*. (1993), should be used to calculate the refractivity at the antenna, N_0^{rad}. This will insure a more accurate calculation of the refraction at the antenna.
- The suggested structure for the pointing coefficient information will allow for a great deal of flexibility in implementation of the ALMA pointing model. Ultimately, these data should be incorporated into a general observatory database to allow detailed tracking of the pointing characteristics of the ALMA antennas.

8 Acknowledgements

Thanks to Bryan Butler for providing the Liebe *et al*. (1993) model, for pointing out many of the references to the atmospheric refractivity literature, including the important Yan (1996) reference, and for some useful comments on an early version of this document. Thanks also to Pat Wallace for his comments on a draft of this document and for many useful discussions.

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