# ALMA Memo 520
# A Refined Method for Estimating Calibrator Counts Above 90 GHz

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**Abstract**

We reanalyze the 90 GHz detections of Holdaway, Owen, and Rupen (1994)) and present a physically plausible model which can be more sensibly extrapolated to higher frequencies. This model predicts fewer calibrator sources at frequencies above 90 GHz than the simple model of Holdaway and D'Addario (2004). Fast switching efficiency will be minimally impacted, though these new source count estimates will often make calibrator observations at 90 GHz more attractive than calibrating at the target freqency.

## 1 Introduction

In fast switching phase calibration, we expect to measure the atmospheric phase on a calibrator every 20-30 s (or longer if water vapor radiometry is successfully applied). Tha calibrator sources are expected to be flat spectrum quasar cores in the 20-200 mJy range, and could be observed at 90 GHz with the phase solutions extrapolated up to the target frequency, or could be observed at the target frequency. To quantify the success of fast switching phase calibration for ALMA, we need to understand the millimeter and submillimeter wavelength source counts of flat spectrum quasars.

Holdaway, Owen, and Rupen (1994) (hereafter HOR) generated a sample of 367 bright flat spectrum quasars selected from 8.4 GHz VLA Array core fluxes (Patnaik *et. al*, 1993), and observed them at 90 GHz with the NRAO 12m telescope. From the detections and upper limits at 90 GHz, and the 8.4 GHz core fluxes, HOR derived a distribution of spectral index between 8.4 GHz to 90 GHz, or $\alpha_{8.4}^{90}$. Flat spectrum source counts at 5 GHz, mainly determined from single dish measurements, were adjusted downward to estimate the contribution from just the core (the distribution of 5 GHz core fraction of these flat spectrum sources was derived from the properties of the flat spectrum members of the 3CR2 sample). After the 5 GHz flat spectrum counts were statistically adjusted to represent just the core emission, HOR then scaled the counts up to 90 GHz using the distribution of $\alpha_{8.4}^{90}$. We use these estimated 90 GHz flat spectrum counts to estimate the density of ALMA fast switching phase calibrators on the sky. In addition, HOR observed 51 steep spectrum quasars with bright cores, and through the spectral index distribution between 8.4 GHz to 90 GHz of these cores, steep spectrum source counts at 5 GHz, and the distribution of core fraction of the steep spectrum members of the 3CR2 sample, were able to estimate that about 10% of the potential millimeter calibrators could be flat spectrum cores of steep spectrum sources.

We also have reason to estimate the source counts of the compact flat spectrum sources at higher frequencies. One scheme for ALMA phase calibration is to perform the fast switching calibrator observations at 90 GHz and to scale the phases up to the target source observing frequency. However, this requires that we periodically observe a second calibrator which is bright at both the target frequency and the calibration frequency (Holdaway and D'Addario, 2004). This second calibrator does not need to be too close to the target source, as it is only used to solve for the cross-band instrumental phase drift. Another scheme for ALMA phase calibration is to perform fast switching at the observing frequency, which could be as high as 950 GHz. Even if we use water vapor radiometry (WVR) to correct for atmospheric phase fluctuations, we will still need to occasionally perform astronomical phase calibration at the target frequency to correct for the instrumental phase drifts. Each of these phase calibration measurements require some bright, compact sources which can be quickly detected at high frequencies. It would be extremely helpful to know the density of flat spectrum quasars as a function of frequency between 90 GHz and 950 GHz. The observational demands of measuring the quasar density in the 10-100 mJy range above 90 GHz are currently prohibitive; this will be one of ALMA's fruits. However, we can estimate the density of flat spectrum quasars above 90 GHz with a clever model based on a reinterpretation of our earlier 90 GHz observations.

### 1.1 Oops! We can do better!

Holdaway and D'Addario (2004) made a rough attempt at estimating the numbers of calibrators above 90 GHz for their calculation of ALMA fast switching efficiency from 35 GHz to 950 GHz. They simply took HOR's distribution of $\alpha_{8.4}^{90}$, assumed a uniform steepening in $\alpha$ of $+0.5$ at 90 GHz (so these sources would be falling in flux faster above 90 GHz), and applied this to the total (flat spectrum sources and steep spectrum sources' cores) estimated 90 GHz calibrator counts from HOR. However, this is not a physically realistic model for what is going on with these sources. Some of these sources with flat spectrums at low frequencies, will have reached the spectral break somewhere between 8 and 90 GHz, which will be reflected in the 8-90 GHz spectral index. Other sources will still be flat spectrum at 90 GHz, and will have a spectral break at higher frequencies. In this work, we seek a model for the high frequency source counts which relies upon a more physically realistic basis than that presented in Holdaway and D'Addario, one which takes a distribution of break frequencies and the physical conditions above and below the break frequency into account.

## 2 The Basis of our Model

In this section, we only deal with flat spectrum quasars. We define the spectral index to be

$$S(\nu) \propto \nu^{-\alpha}.$$

The canonical theory of flat spectrum radio sources indicates that sources will have $\alpha = 0$ at low frequencies where they are optically thick. As the observing frequency increases, the point at which the jet transitions from optically thick to optically thin moves closer to the quasar's central engine, until at some break frequency $\nu_b$ the source becomes optically thin, and the spectrum turns over with $\alpha = 0.8$, the spectral index of optically thin synchrotron emission. A simple model for the flux of such a source, based on this picture, is described by

$$S(\nu) = S(\nu_\circ) \left(\frac{\nu_b}{\nu_\circ}\right)^{-\alpha_f} \left(\frac{\nu}{\nu_b}\right)^{-\alpha_s},$$

where $\nu_\circ$ is some fiducial frequency (in our case, 5 GHz), $\alpha_f$ is the flat spectrum, or optically thick, spectral index, which is about 0.0, and $\alpha_s$ is the steep spectrum, or optically thin, spectral index, which is about 0.8.

In Figure 1 we depict the distribution of $\alpha_{8.4}^{90}$ derived from the observations of HOR. This distribution, which included non-detections at 90 Ghz as 3-sigma upper limits to the 90 GHz flux, was derived using the *ASURV* code (Feigelson and Nelson, 1985). This distribution looks like a a superposition of two Gaussians, so we tried fitting two Gaussians. The boxes represent the integral of the sum of the two Gaussians over each histogram bin, and the goodness of the fit was judged by the $\chi^2$ between the histogram bin height and the boxes. This decomposition turns out to be very illuminating.

The Gaussian on the left plausibly represents flat spectrum Quasars which have not turned over by 90 GHz. This Gaussian is centered at $\alpha = -0.03$, which is very close to 0.0. It has a width of 0.142, and represents 19% of the sources.

The larger Gaussian on the right plausibly represents flat spectrum Quasars which have turned over between 8.4 and 90 GHz. This Gaussian is centered at $\alpha = 0.431$, has a width of 0.280, and represents 81% of the sources. The left, flat spectrum end of this second, wider Gaussian would be populated by sources which turned over closer to 90 GHz, and the right, steep spectrum end of this distribution would be populated by sources which had turned over closer to 8.4 GHz.

We can combine this interpretation of the Gaussian decomposition with the simple physical picture painted in the beginning of this section. The data indicate that physical reality won't give us delta function distributions of spectral index at 0.0 and 0.8. There will be some intrinsic variation in spectral index due to flares which appear first at high frequencies, but then migrate out along the jet and gradually are seen at the low frequencies. So, we do not expect all flat spectrum sources to have a spectral index of 0.0. The small, left-side Gaussian fit in Figure 1, which we have associated with flat spectrum sources which have not turned over, tells us the intrinsic spread in spectral index that these sources display.

Also, the optically thin spectral index must have some scatter about the theoretical value of 0.8. The data require it as well, because the larger Gaussian has some spectral index values as high as 1.3. This Gaussian representing sources which have turned over is much wider than the intrinsic spread because it includes some sources which turned over close to 8 GHz (which will have larger spectral index) and some sources which turned over close to 90 GHz (which will have smaller spectral index).

So, our model has become a bit complicated: at low frequencies, our sources have an intrinsic spread of spectral index about 0.0, which is given by the Gaussian of width 0.142. At high frequencies, sources which have turned over to opticakkly thin have an intrinsic spread of spectral index about the value 0.8, which we *assume* to be also a Gaussian of width 0.142. We also have a distribution of turnover frequencies ranging from 8.4 GHz to 90 GHz to reproduce the large Gaussian in the spectral index distribution decomposition. And last, the 19% of the sources which have not turned over by 90 GHz will have a break frequency somewhere above

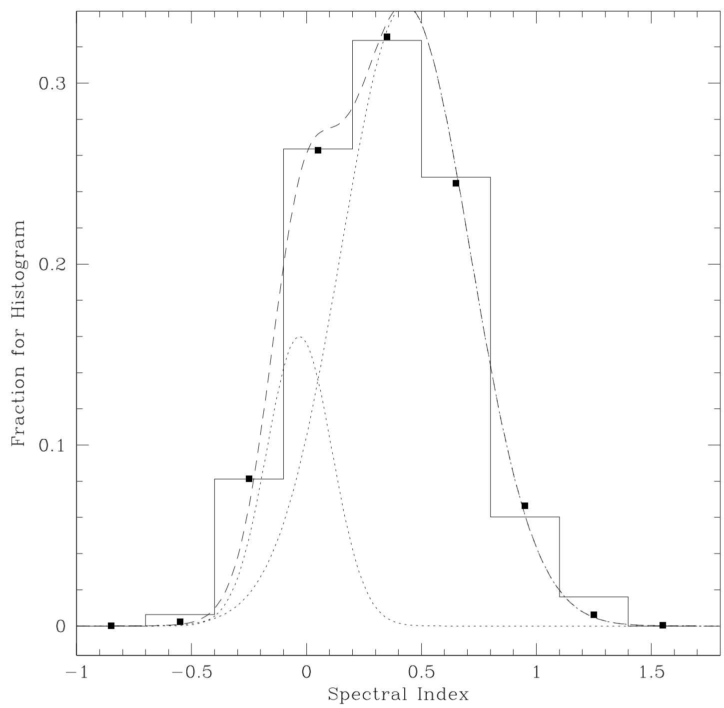


Figure 1: The distribution of spectral index between 8.4 and 90 GHz as derived from the observations of Holdaway, Owen, and Rupen (1994) using Feigelson's *asurv* package, and a best fit decomposition into two Gaussians. These two Gaussians represent flat spectrum Quasars which have not turned over yet (small Gaussian on the left), and the population of Quasars which have turned over between 8.4 and 90 GHz (the large Gaussian on the right).

90 GHz, but we don't know where; we will be able to make assumptions about what these sources do which will provide an upper limit, a lower limit, and a "best guess" scenario to the high frequency source counts.

## 2.1 Distribution of Break Frequencies

Determining the distribution of break frequencies is a classic inverse problem. If we knew the distribution of break frequencies, then using a Monte Carlo method, we could make a population of sources with a spectral index $\alpha_{8.4}^{\nu_b}$ distribution consistent with the Gaussian centered on 0.0 and of width 0.142, and a spectral index $\alpha_{\nu_b}^{90}$ distribution consistent with a Gaussian centered on 0.8 and also of width 0.142, and calculate the distribution of spectral index from 8.4 to 90 GHz. We have the opposite problem: we know the final distribution of $\alpha_{8.4}^{90}$ (ie, the wide Gaussian to the right of Figure 1), and we assume the distributions of spectral index below and above the break, and we need to solve for a plausible distribution of break frequencies which results in the distribution of $\alpha_{8.4}^{90}$.

The distribution of break frequencies we tried which had the best fit to the Gaussian distribution of $\alpha_{8.4}^{90}$ was a series of equal magnitude delta functions placed at 8.4, 12, 17, 21.5, 28, 38, 50, and 88 GHz. While this is not a very plausible form for the break frequency distribution, it convincingly illustrates that we need some sources at both the 8.4 GHz end and the 90 GHz end in order to make a distribution of $\alpha_{8.4}^{90}$ broad enough to reproduce the desired Gaussian. It also illustrates that the distribution is approximately flat in logarithmic bins..

These facts hint at a simple model for the distribution of break frequencies: $\psi(\nu_b) \propto \nu^a$. One advantage of such a model is that it extends beyond 90 GHz, so we might guess at the break frequency distribution of the remaining 19% of sources which have not turned over by 90 GHz. The best fit value of the power law exponent $a$ is -1.13, and recovers the desired distribution of $\alpha_{8.4}^{90}$ with an error about three times larger than the collection of delta functions. However, this model is simple and perhaps more reasonable, having only one parameter (as opposed to the eight parameters of the delta function model). Furthermore, if this distribution is extrapolated above 90 GHz, it will account for the remaining 19% of sources if it continues out to 181 GHz. We will consider this power law distribution to be the "best guess" break frequency.

A lower limit to the high source flat spectrum quasar source counts could be made by assuming the 19% of still-flat spectrum sources all have a break frequency of 90 GHz, and an upper limit can be made by assuming these sources don't turn over below ALMA'shighest observing frequency, 950 GHz.

## 3 Handling The Computations

OK, we have Gaussian distributions of the optically thick spectral index (a Gaussian of width 0.142 centered at 0.0), the break frequency ($\psi(\nu_b) \propto \nu^{-1.13}$), and the optically thin spectral index (a Gaussian of width 0.142 centered at 0.8). Furthermore, we have 5 GHz source counts derived mostly from single dish observations which will include extended emission, which we augment with a distribution of core flux fractions derived from flat spectrum 3CR2 sources (as in HOR). From this collection of distribution functions, we can estimate the source counts at any frequency above 5 GHz.

In our first attempt to extrapolate the source counts to higher frequencies (Holdaway and D'Addario, 2004), we applied the distribution of spectral index $\alpha_{8.4}^{90}$ to the estimated 90 GHz source counts of flat spectrum sources plus the cores of steep spectrum sources calculated in HOR. This was computationally easy, similar to the calculations we had performed in the past.

However, the more physically realistic model presented in this memo requires that we handle the distribution of 5 GHz flat spectrum fluxes, along with distributions in optically thin spectral index, turnover frequency, and optically thick spectral index. The way we deal with all these distributions is to simulate hundreds of thousands of sources, each with its own 5 GHz core flux, low frequency spectral index close to 0.0, break frequency between 8.4 GHz and 181 GHz, and high frequency spectral index. Hence, for each simulated source we can calculate the flux at an arbitrary frequency, and we then look at the number counts as a function of flux for these simulated distributions. At the high flux end, there will be many fewer sources, and the statistics will start to get noisy, which can be seen in Figures 2 and 3. The first figure shows the best guess estimate of the source counts assuming the 19% of still-flat sources have the same break frequency distribution as the other sources which have already turned over. The second figure shows the best guess counts plus the upper and lower limits, obtained by assuming those 19% of sources never turn over, and by assuming they all turn over at 90 GHz.

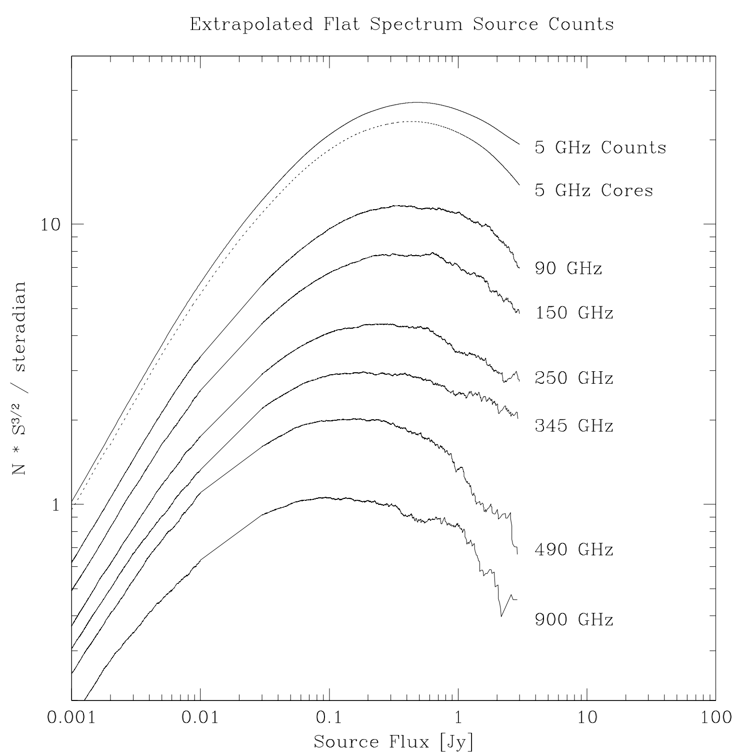


Figure 2: The measured 5 GHz flat spectrum source counts, the estimated 5 GHz counts of flat spectrum cores, and estimated counts for 90, 150, 250, 345, 490, and 900 GHz, assuming the power law distribution of turnover frequencies.

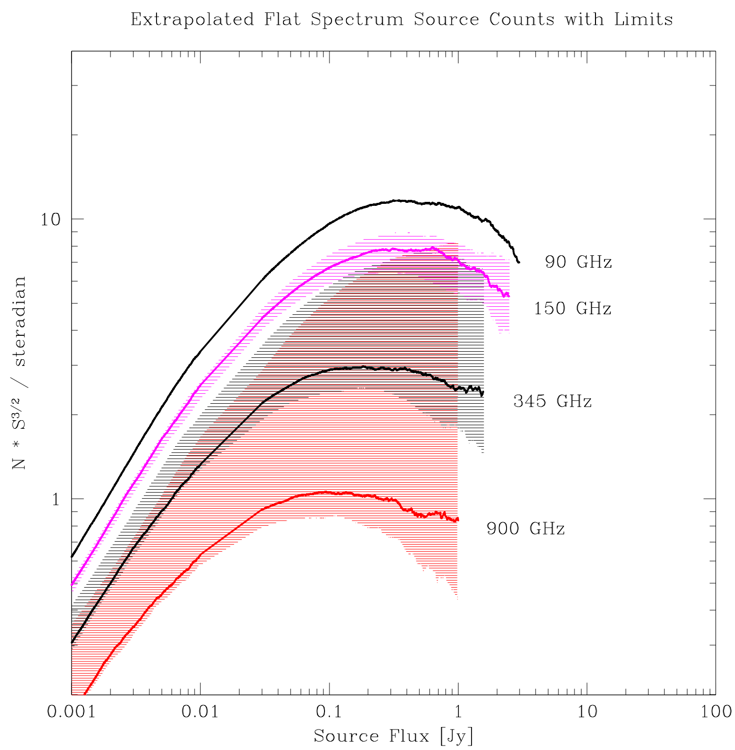


Figure 3: Upper and lower limits for source counts at 150, 345, and 900 GHz. The lower limit is obtained by assuming the 19% of still-flat spectrum sources all turn over at 90 GHz. The upper limit is obtained by assuming those 19% of still-flat sources *never* turn over. The solid line in the center of each range represents our "best guess" of what the source counts do. The "best guess" is obtained when we continue the best fit power law distribution of break frequencies up to 181 Ghz (the frequency required to make all the sources turn over). The upper limit becomes less meaningful as the frequency increases.

## 4 Contributions from Steep Spectrum Quasars?

In the above analysis, we have only considered flat spectrum quasar counts. HOR found that at 90 GHz, the flat spectrum cores of steep spectrum quasars made a small contribution to the potential calibrators - a bit less than 10%. However, in the unified model of extragalactic radio sources, the steep spectrum sources are drawn from the same population as flat spectrum source, but the flat spectrum sources have cores which are relativistically beamed toward us, thereby boosting the flux of the cores well above the steep spectrum emission. If the cores of steep and flat spectrum objects are the same types of objects but the flat spectrum cores are beamed toward us (ie, blue-shifted), we expect that the break frequencies of the cores of steep spectrum objects to be at lower frequencies than the cores of flat spectrum sources, and we would expect the spectral index distribution of the cores of steep spectrum sources to be steeper than the spectral index distribution of the cores of flat spectrum objects. However, the spectral index distributions in HOR is the opposite of what we expect, with the cores of steep spectrum objects being more flat and inverted than the flat spectrum counterparts. From the data in HOR, we can't really say much about the break frequency of the cores of the steep spectrum sources, but we expect the contribution from these sources to be minimal at high frequencies.

## 5 Comparison with Holdaway and D'Addario (2004)

Comparing these new source count estimates between 90 GHz and 900 GHz with those of Holdaway and D'Addario (2004), we see that the current results have somewhat fewer sources at 90 GHz (because the cores of steep spectrum sources were not included). At higher frequencies, the current work's counts are significantly lower than those in Holdaway and D'Addario. If we are calibrating at 90 GHz, the new source count estimates will result in only a minor decrease in fast switching efficiency as compared with the results in Holdaway and D'Addario (of order less than 1%, because sensitivity on the 90 GHz calibrator source is not at all the limiting factor). One result from Holdaway and D'Addario is that fast switching calibration at the target frequency will work fine up to 350 GHz, but that at higher frequencies, calibrating at 90 GHz and scaling the solutions up to the target frequency made an increase in efficiency of about 5% (ie, from 70% to 75%). The rather large decrease in the high frequency counts presented in this paper would affect fast switching simulations by a) reducing the observing frequency at which it becomes advantageous to calibrate at 90 GHz, perhaps to as low as 230 GHz, and b) increase the efficiency savings by calibrating at 90 GHz for the higher target frequencies. One last effect that these new source count estimates would have on fast switching is that there would be fewer high frequency bright sources we could use to measure the cross-band phase drift, which would also slightly decrease the calibration efficiency at high frequencies.

Clearly, we should revisit the calculations of Holdaway and D'Addario (2004) to fully consider the implications of these new source count estimates.

## 6 References

Feigelson, E.D., and P.J. Nelson, 1985, ApJ, 293, 192.

Holdaway, M.A., and D'Addario, 2004, "Simulation of Atmospheric Phase Correction Combined with Instrumental Phase Calibration Using Fast Switching", LAMA Memo 803.

Holdaway, M.A., F.N. Owen, and M.P. Rupen, 1994, " Source Counts at 90 GHz", ALMA Memo 123.

Patnaik et al, 1993, MNRAS 261, 435.